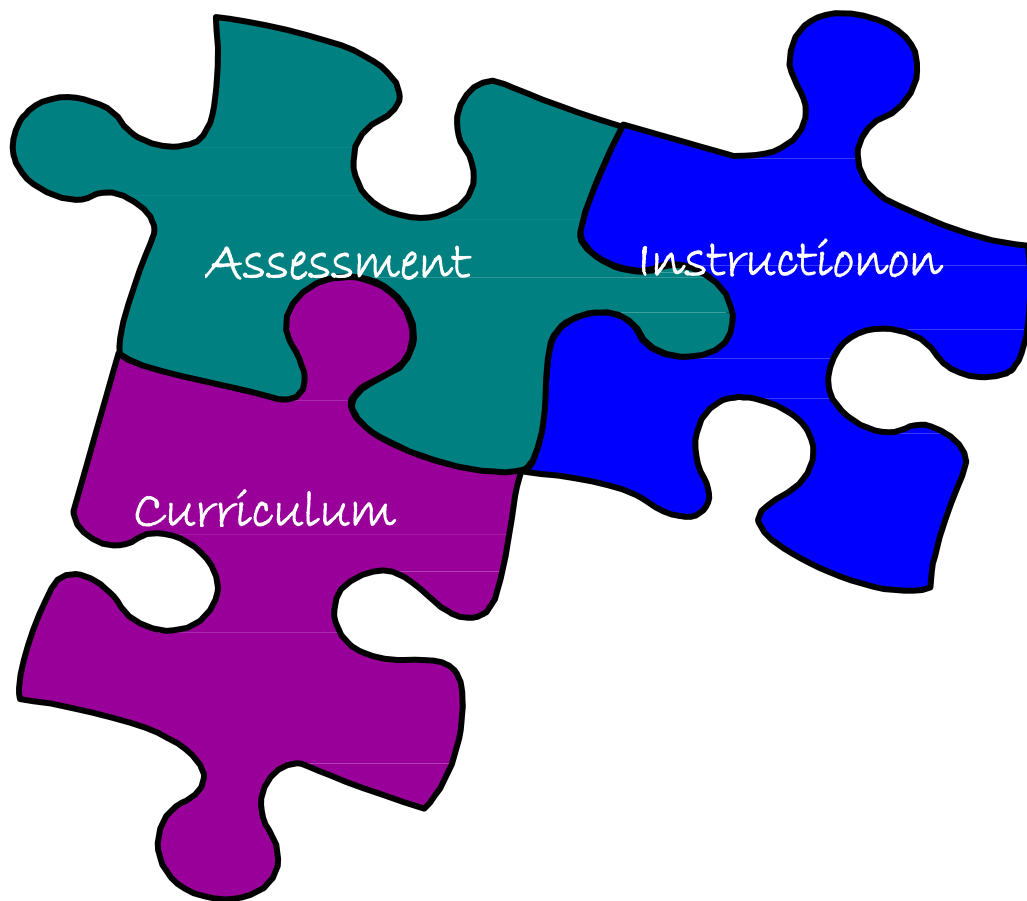
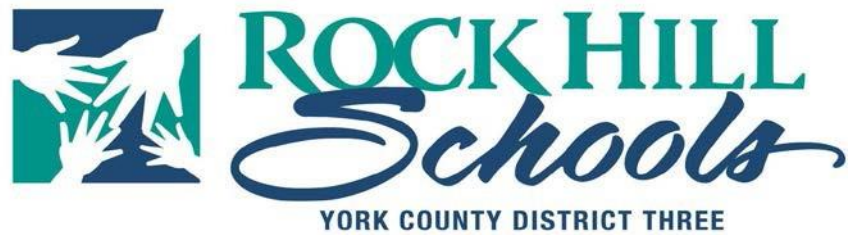


# Math Expectation Guide



*"Putting the pieces together for success  
in mathematics!"*



## **Math Expectation Guide**

# **Kindergarten through Grade 5**

**Dr. Kelly Pew, Superintendent**

**Dr. Harriet Jaworowski, Associate Superintendent for  
Accountability and Instruction**

**Updated 2016**



## **Mathematics Instruction in Rock Hill Schools Kindergarten through Grade 5**

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The committee began meeting in the spring of 2008 and continued through the fall of 2008 and early spring of 2009. Once the draft was complete, input was solicited from all committee members and the review team listed above. All comments and recommendations were considered before publishing this draft.

## Preface

The Mathematics Expectation Guide is a reference document, that has been prepared to guide educators through the processes that have been deliberately designed to improve student achievement and close achievement gaps. This guide is a resource for teachers not a curriculum. Research shows that teachers directly affect student learning through the design of work that has those qualities that are most engaging to students. Therefore, our instructional goal is to increase, enhance, and meet the academic needs of all students through authentic assessment, standards-based curriculum, and engaging instruction.

This guide revolves around differentiated instruction and learning through an inquiry-based approach. It consists of 3 main sections: assessment, curriculum, and instruction. Each section operates interdependently of the other in order for students to attain their greatest mathematical potential.

To get started, there are a number of strategies and tools that need to be put in place to find out specifics about students. This can be done through the interpretation and evaluation of several different types of assessment and some of these assessment strategies should be on-going throughout the instruction process. In essence, in order to close the achievement gap and increase student achievement, we, as educators, need to understand that assessment assists in determining where to begin instruction. To promote equity, students should have many opportunities to demonstrate THEIR understanding of math concepts and skills and should be given immediate feedback as teachers monitor and reflect on the learning process of students. Assessment is also a valuable tool for teachers to use as they make decisions regarding instruction while continuing to meet the academic needs of all students.

Curriculum is the part of this equation that determines what teachers are to teach as they facilitate students and encourage them to take ownership of their learning. This reference guide provides teachers with a framework of clear and concise explanations of vertically aligned standards as well as sample item representations. The curriculum framework can also be used to differentiate instruction, create flexible groupings, and develop common assessments.

Instruction should be authentic, engaging, and differentiated to meet the academic needs and interests of students through the mathematical processes that develop students' abilities to think critically, problem solve, make connections, communicate their ideas, represent what they have learned in various ways, and make reasonable estimations and provide proof of solutions.

It is our intention that teachers will use this guide to meet the ever demanding and challenging mathematical needs of students as we work towards increasing student achievement and closing the achievement gap.

The Rock Hill School District Professional Code: *Put Students First, Nurture Relationships, Work Together for a Shared Vision, Grow Professionally, Continuously Find Ways to Improve*  
*Learn+Grow+Connect+Thrive*

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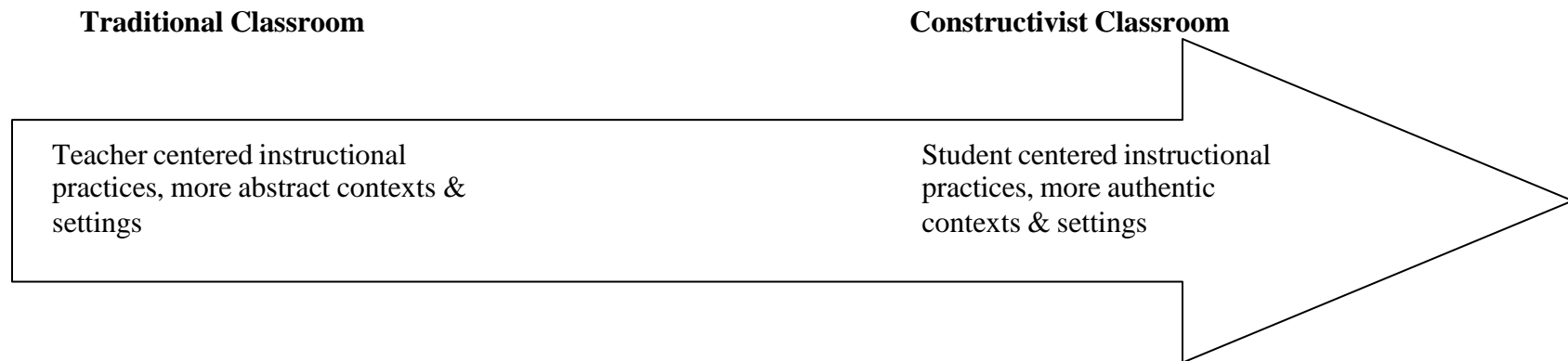
- Grid Paper
- Helpful Websites

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# Traditional Classroom vs. Constructivist Classroom

Through the support of the Rock Hill Schools' Instruction Department and this Math Expectation Guide, mathematics instruction should transition from the traditional classroom approach to learning to the constructivist classroom approach to learning. The teacher's role is not to transmit knowledge to the students, but to create an environment in which students actively explore mathematical ideas through standards-based, engaging investigations. As students investigate and solve problems, important mathematical ideas and concepts will be learned with understanding, not just through memorization. The student's role is to be actively involved in doing mathematics, thinking and reasoning about mathematics, searching for answers and ultimately, taking responsibility for their own learning.



<http://www.coe.uga.edu/ctl/theory/external.pdf>

The figure on the following page was modified by Shake Seigel from “**Constructivism as a Paradigm for Teaching and Learning**”.  
<http://www.thirteen.org/edonline/concept2class/constructivism/index.html> July 2004



# Traditional Classroom vs. Constructivist Classroom

## **Traditional Classroom**

Curriculum begins with the parts of the whole.  
Emphasizes basic skills.

Strict adherence to fixed curriculum is highly valued.

Materials are primarily textbooks and workbooks.

Learning is based on repetition.

Teachers disseminate information to students; students are recipients of knowledge.

Teacher's role is directive, rooted in authority.

Assessment is through testing, and correct answers are expected.

Knowledge is inert.

Students work primarily alone.

## **Constructivist Classroom**

Curriculum emphasizes big concepts, beginning with the whole and expanding to include the parts.

Pursuit of student questions and interests is valued.

Materials include primary sources of material and manipulative materials.

Learning is interactive, building on what the student already knows.

Teachers have a dialogue with students, helping students construct their own knowledge.

Teacher's role is interactive, rooted in negotiation.

Assessment includes student works, observations, and points of view, as well as tests. Process is as important as product.

Knowledge is seen as dynamic, ever changing with our experiences.

Students work primarily in groups.

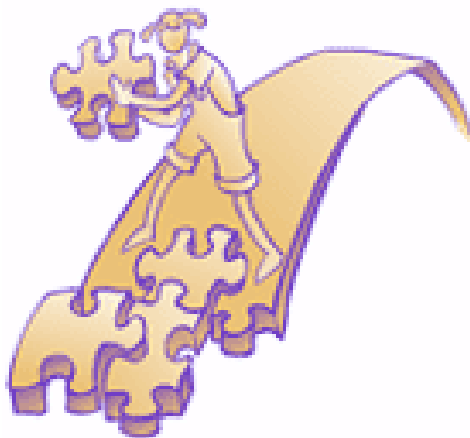


## Teaching and Learning Mathematics with Understanding

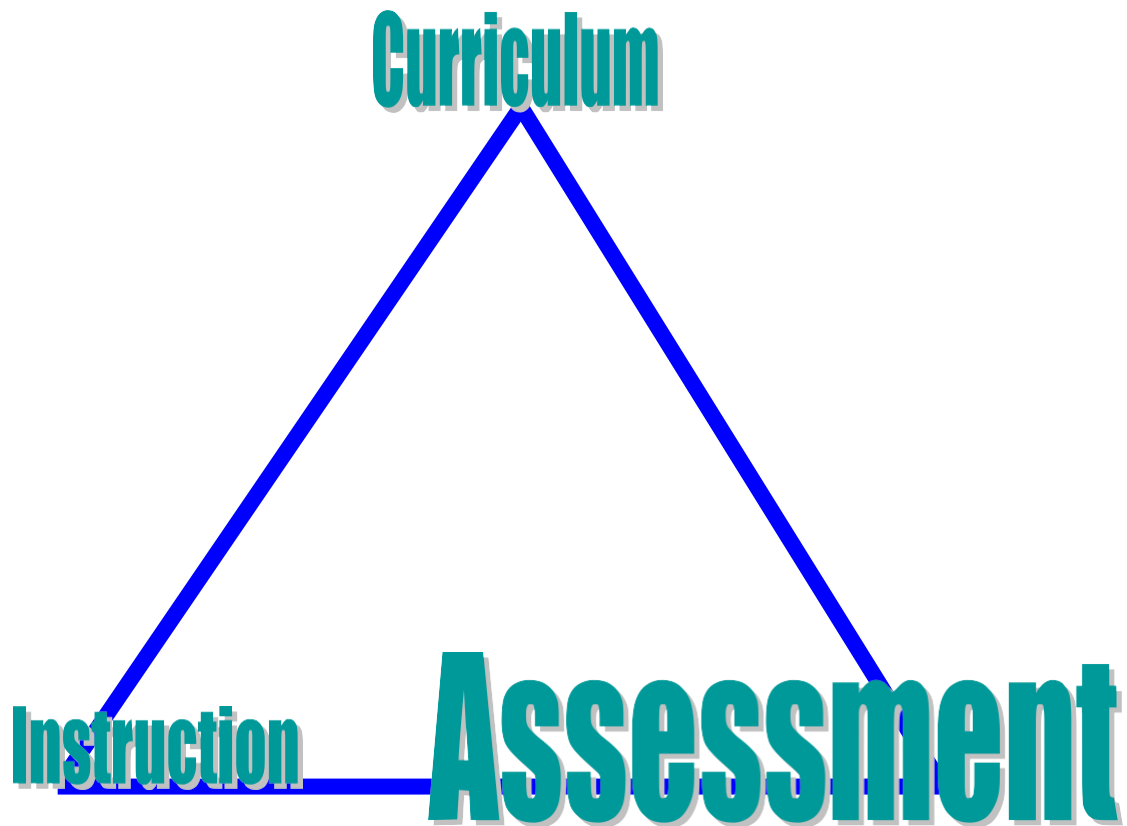
The table outlines the five important dimensions of a classroom that promotes teaching and learning mathematics with understanding and the core features regarding each dimension.

Adapted from: Hiebert et al. *Making Sense: Teaching and Learning Mathematics with Understanding*

DIMENSIONS	CORE FEATURES
<b>Nature of Classroom Tasks</b>	<ul style="list-style-type: none"> <li>• Make mathematics problematic</li> <li>• Connect with where students are</li> <li>• Should be interesting/engaging</li> <li>• Should encourage communication and reflection</li> <li>• Should allow students to use tools</li> <li>• Leave behind something of mathematical value</li> </ul>
<b>Role of the Teacher</b>	<ul style="list-style-type: none"> <li>• Select tasks with goals in mind</li> <li>• Facilitate, not direct, learning</li> <li>• Encourage a variety of strategies</li> <li>• Encourage mathematical conversations</li> <li>• Differentiate instruction</li> <li>• Establish a mathematical community of learners</li> </ul>
<b>Social Culture of the Classroom</b>	<ul style="list-style-type: none"> <li>• Ideas and methods are valued</li> <li>• Students choose and share their methods</li> <li>• Collaboration and communication are essential</li> <li>• Mistakes are learning sites for everyone</li> <li>• Correctness rests in mathematical argument – not the teacher</li> </ul>
<b>Mathematical Tools as Learning Supports</b>	<ul style="list-style-type: none"> <li>• Meaning for tools must be constructed by each user</li> <li>• Used with purpose – to solve problems</li> <li>• Used for recording, communicating, and thinking</li> </ul>
<b>Equity and Accessibility</b>	<ul style="list-style-type: none"> <li>• Tasks are accessible to all students</li> <li>• Tasks reflect consideration of diverse cultures</li> <li>• Every student contributes and every student is heard</li> </ul>



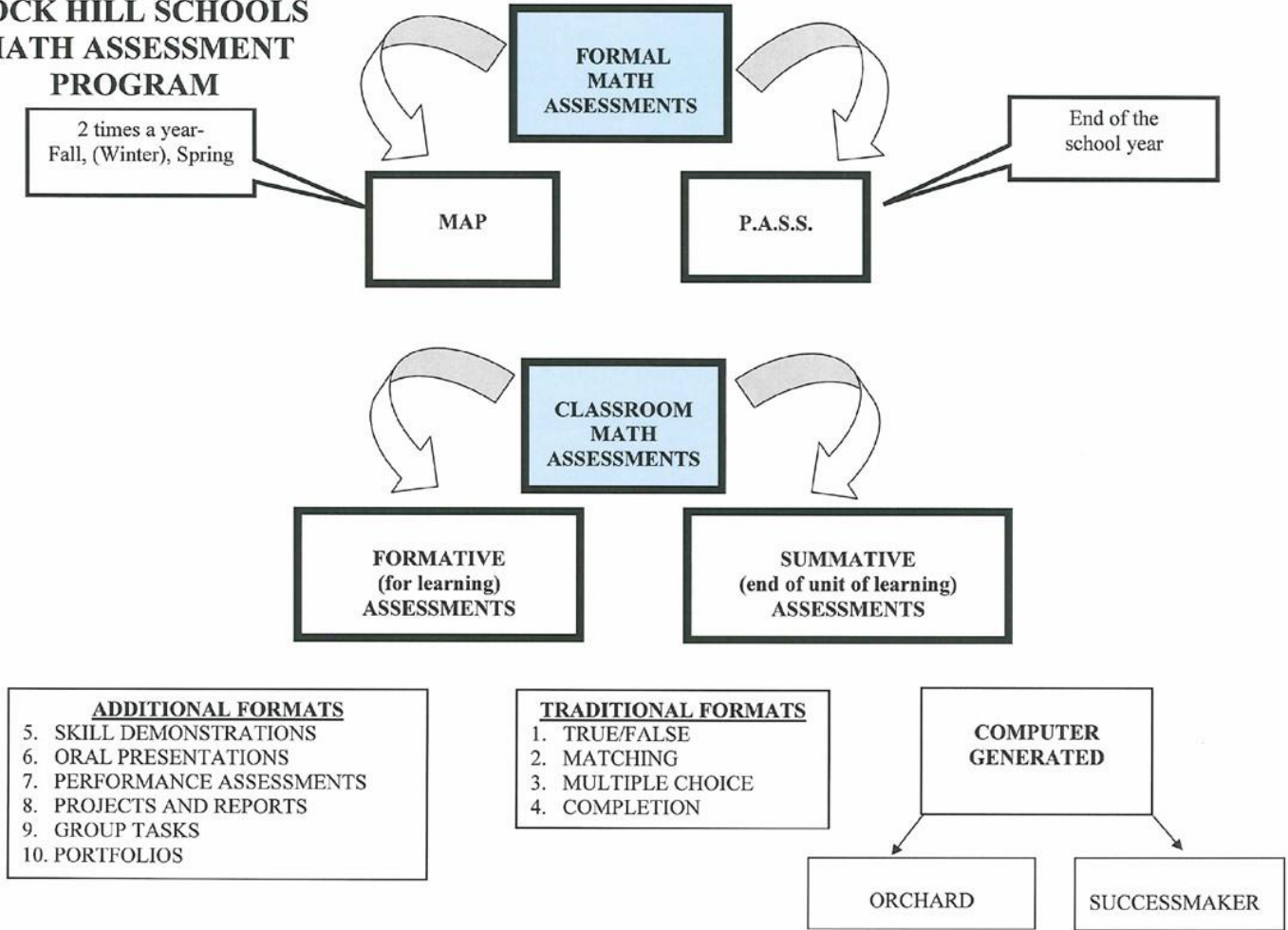
# Assessment



*“And while assignments and quizzes are important, merely checking whether or not answers are correct is insufficient. Assessments must also uncover what students understand and provide insights into how they think and reason. Key to assessing students’ math learning is to delve into how students arrive at answers.”*

*Marilyn Burns*

**ROCK HILL SCHOOLS  
MATH ASSESSMENT  
PROGRAM**



# Assessment

## Mathematical assessments should:

- Be an integral part of instruction and should enhance student learning.  
Assessments that enhance mathematics learning become a routine part of instruction.

Such assessments incorporate activities that are consistent with, and sometimes the same as, the activities used in instruction.

- Promote equity.  
In an equitable assessment, each student has an opportunity to demonstrate his/her understanding of mathematics.

Assessments should allow for multiple approaches because different students show what they know and can do in different ways.

- Be a tool for monitoring student progress and evaluating student achievement.  
As teachers monitor student progress and achievement, they should focus on student understanding as well as procedural skills.

As students monitor their own progress, the teacher's feedback on the assessments should be constructive and focused to help the students understand his/her mistakes and ways to improve.

Assessments of isolated facts and skills should not be emphasized above assessments of conceptual mathematical understandings.

- Be a valuable tool for making instructional decisions  
When teachers use assessment to make instructional decisions they can make instruction more responsive to students' needs.

Teachers should engage in ongoing analysis of teaching and learning by observing, listening to, and gathering information about students to assess what they are learning and the effects of the instructional tasks presented (NCTM).

# Types of Assessments

**Summative Assessments** - Summative assessments are final assessments given periodically to determine at a particular point and time what students know and do not know.  
Examples of summative assessments:

**State Standardized Assessment**

**District benchmark or interim assessment - MAP**

**Performance-based assessment** - A well-defined task is identified and students are asked to create, produce, or do something, often in settings that involve real-world application of knowledge and skills.

**Post- Test/ Unit Test**

**Formative Assessments** - Formative assessments are the part of the instructional process that provides the information needed to adjust teaching and learning while it is happening. Formative assessment is “for” learning, it informs both teachers and students about student understanding. Examples of formative assessments:

**Pre-tests, quizzes, and post-tests**

**Math journals and portfolios**

**Academic prompts**

**Observations/ Anecdotal records**

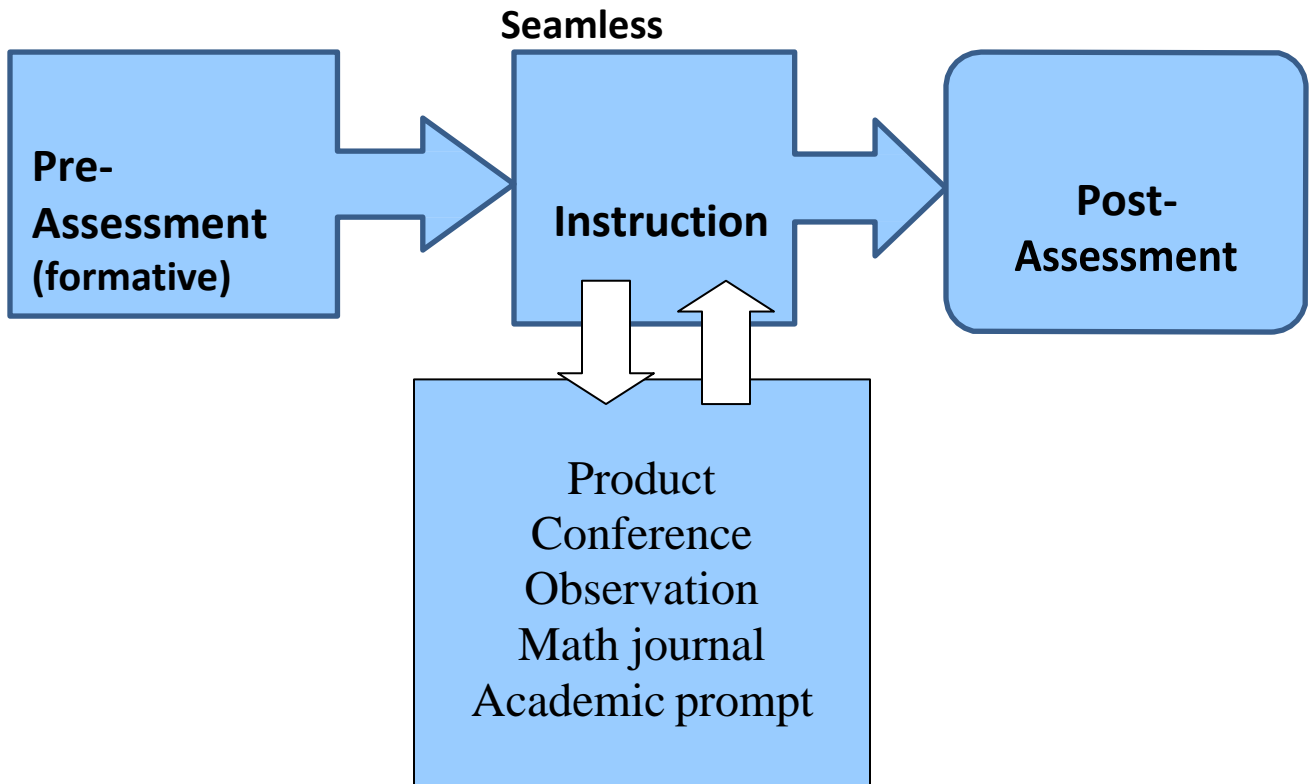
**Conferences/Interviews**

**Exit Slips/Tickets to Transition (Leave)**

## Balancing Assessment

In a balanced assessment program of summative and formative assessments, teachers are able to gather information that is an integral part of the learning process for students and teachers. To better understand what students have learned, teachers need to consider information from tests, products created, observational notes, student led conferences, and communication among students and teachers. The more teachers know about individual students as they engage in the learning process, the better they can adjust instruction to ensure that all students continue to achieve progressively toward the learning goal.

### Making Formative Assessments and Summative Assessment



To design instruction to meet the needs of students, instruction and assessment should be intertwined in the unit's lessons. Formative assessments should be embedded to inform instruction.

The teacher administers a pre-test to determine what the students know about the concept. After giving the pre-test, several formative assessments should be presented or incorporated into the teaching. The formative assessments give the teacher immediate feedback about the students' understanding of the concept. The teacher is able to monitor and adjust instruction, so students can receive remediation, if necessary, or can be further challenged. The goal is for students to have opportunities to practice and master concepts and learning objectives before being given the post test.

The forms (observation, product, conference, etc.) of assessment can be used as formative or summative assessment tools. The design of the assessment determines the type of feedback the teacher will receive.



# Common Formative Assessments

## What Are Common Formative Assessments?

- Periodic assessments collaboratively designed by grade-level teams of teachers.
- Designed as matching pre- and post-assessments to ensure same-assessment comparisons of student growth.
- Similar in design and format to district and state assessments.
- Should be a blend of item types, including selected-response (multiple choice, true/false, matching), and constructed-response (short- or extended).
- Student results should be analyzed to guide instructional planning and delivery.

## Guidelines for Designing Common Formative Assessments:

- Identify standard/indicators for your grade level.
- Create no more than one to five learning targets for that assessment.
- “Unwrap” the standards for the concepts students need to know and be able to do.
- From those “unwrapped” standards, determine Big Ideas that represent the integrated understanding students need to gain.
- Determine the level of rigor and the level of understanding (using the indicators’ verb) to assess the concepts.
- Collaboratively design common formative pre- and post-assessments - aligned to one another - that assess students’ understanding of the concepts, skills, and Big Ideas from the “unwrapped” standards.
- If the concept is new learning for your grade level, the pre-test should include prerequisite skills.
- Include both selected-response and constructed-response items.
- Guarantee that each target receives enough of a sampling to certify learning (generally five to ten questions per target area).
- Review items to determine if student assessment results will provide *evidence of proficiency* regarding the standards in focus; modify items as needed.

## Benefits of Using Common Formative Assessments:

- Regular and timely feedback regarding student attainment of most critical standards. This allows teachers to modify instruction to better meet the diverse learning needs of all students
- Multiple-measure assessments that allow students to demonstrate their understanding in a variety of formats
- Ongoing collaboration opportunities for grade-level, course, and department teachers
- Consistent expectations within a grade level, course, and department regarding standards, instruction, and assessment priorities
- Agreed-upon criteria for proficiency to be met within each individual classroom, grade level, school, and district
- Deliberate alignment of classroom, school, district, and state assessments to better prepare students for success on state assessments

- Results that have predictive value as to how students are likely to do on each succeeding assessment, in time to make instructional modifications

*Source: Larry Ainsworth & Donald Viegut, Common Formative Assessments: How to Connect Standards-based Instruction and Assessment (Corwin Press, 2006).*

# Building and Using Formative Assessments

## **What should teachers do with the information collected from assessments?**

After teachers have planned and gathered information collected from pre-assessments and /or other forms of data, the information should be interpreted to determine how the data is to be utilized to enhance student learning. In essence, teachers should be using several sources from which to evaluate student achievement.

## **Teachers should provide immediate feedback by:**

- Differentiating instruction or presenting instruction in a new way
- Planning instruction that is challenging and engaging using the standards through enrichment and performance-based problems or tasks
- Focusing on concepts/ skills that students are having difficulty with
- Collaborating with other teachers who might experience similar challenges
- Continuing to monitor student progress
- Praising students for current accomplishments

## **Key points for pre- and post assessments:**

- Pre- and Post assessments must assess the same objectives.
- Pre- and Post assessments should include strategies which will clearly identify students' strengths and weaknesses (from computation to problem solving and from concrete to abstract).
- Pre- and Post assessment methods should be congruent with the learning objectives.
- Pre- assessments should be designed to give immediate feedback.
- Pre- assessments should be analyzed for driving instruction, but not graded and recorded in the grade book.
- Pre- assessments should give students a snapshot of their ability to apply certain skills or concepts. (Reassure students that they may not know all of the answers.)
- Pre- assessments should be a working document for students.
- Post- assessments should be graded; however, if the student did not show mastery on the post- assessment, students should have on-going opportunities to meet the targeted learning goal.

(paraphrased from) <http://www-personal.umich.edu/~espring/bestPractices/pretest.html> and <http://www.coedu.usf.edu/ctc/4.HTM>

## **Summative and Formative Assessments can be either formal or informal.**

### **Formal assessments**

- Have data which support the conclusions made from the test. These types of tests are usually referred to as standardized measures.
- Tests that have been tried before on students and have statistics which support the conclusion such as the student is performing below average for his age.
- The data is mathematically computed and summarized. Scores such as percentiles, stanines, or standard scores are most commonly given from this type of assessment.
- Formal or standardized measures should be used to assess overall achievement, to compare a student's performance with others at their age or grade, or to identify comparable strengths and weaknesses with peers.
- Most formal assessments are also **summative** in nature.

### **Informal assessments**

- Assessments that are not data driven but rather content and performance driven.
- Informal assessments should be used to inform instruction.
- The most effective teaching is based on identifying performance objectives, instructing according to these objectives, and then assessing these performance objectives.
- Are sometimes referred to as criterion-referenced measures or performance-based measures.

<http://content.scholastic.com/browse/article.jsp?id=4452>

[http://www.associatedcontent.com/article/918027/how\\_to\\_compare\\_formal\\_and\\_informal.html?cat=4](http://www.associatedcontent.com/article/918027/how_to_compare_formal_and_informal.html?cat=4)

# **Establishing a Baseline for Math Instruction at the Beginning of the School Year**

A pre-assessment should be used to guide instruction and to identify what students know and what they need to know.

Below are a range of data sources and pre-assessment tools that should be used to establish a baseline for math instruction at the beginning of the school year.

## **Permanent Records**

- Report cards
- Test score data

## **Measure of Academic Progress (MAP) – Grades K-8**

- MAP data indicate which students have met the benchmark for the grade and which students have not yet learned the grade-level material.

## **Formal Pre-assessment**

- Pre-tests

## **Informal Pre-Assessment**

- Inventories and Surveys
- Anecdotal Records
- Checklists

# Measure of Academic Progress (MAP)

## MAP is:

- A computerized adaptive assessment that measures the students' ability levels in the five strands of mathematics.
- Administered to students in kindergarten through grade 8

## Information MAP data provides for teachers:

- MAP data indicates which students are meeting their projected average yearly growth goals, and which students are not meeting growth goals.
- Defines flexible groups for instruction.
- Guides differentiated instruction.
- Links test results to skills and concepts included in state standards.
- MAP tests provide highly accurate results that can be used to:
  - Identify the skills and concepts individual students have learned.
  - Diagnose instructional needs.
  - Monitor academic growth over time.
  - Make data-driven decisions at the classroom, school, and district levels.
  - Place new students into appropriate instructional programs.

## How the data informs the teacher's instruction:

- It provides useful information about where a student is learning and guides differentiated instruction.
- It provides information regarding a student's strengths and areas for improvement.
- Student growth and achievement status are both reported, so that teachers can make informed decisions about remediation and enrichment opportunities.

## Information obtained from MAP data

MAP data can give you an indication of math levels and ranges that exist within your class and the number of students that fall within each of those math levels.

There are many reports that can be generated by classroom teachers to be used for instructional guidance. Various reports give you an indication of math levels and ranges that exist within your class and the number of students that fall within each of those math levels. There are also reports that can be generated to share student progress with individual students and parents.

For a complete list of reports available and their purpose:

[https://teach.mapnwea.org/assist/help\\_map/ApplicationHelp.htm#UsingTestResults/MAPReportsFinder.htm%3FTocPath%3D\\_\\_\\_\\_\\_3](https://teach.mapnwea.org/assist/help_map/ApplicationHelp.htm#UsingTestResults/MAPReportsFinder.htm%3FTocPath%3D_____3)

Log on to the NWEA website [teach.map.nwea.org](https://teach.map.nwea.org)

## Authentic Assessment: A New Approach to Assessment

With a new approach to assessment there is a shift in content, learning, teaching, evaluation/ assessment, and expectation.

<b>Towards</b>	<b>Away From</b>
<i>Content:</i> Rich variety of mathematical topics and problem situations	<i>Content:</i> Just arithmetic
<i>Learning:</i> Investigating problems	<i>Learning:</i> Memorizing and repeating
<i>Instruction:</i> Questioning and listening	<i>Instruction:</i> Telling
<i>Evaluation:</i> Several sources judged/ evaluated by teacher	<i>Evaluation:</i> Single test judged externally
<i>Expectation:</i> Concepts and procedures to solve problems	<i>Expectation:</i> Mastering isolated concepts and procedures

### Major shifts in assessment practices:

<b>Towards</b>	<b>Away From</b>
Assessing students' fullest mathematical ability	Assessing only students knowledge of specific facts and isolated skills
Comparing students' performance with established criteria	Comparing students' performance with that of other students
Giving support to teachers and credence to their informed judgment	Designing "teacher-proof" / ready-made assessment system
Making the assessment process public, participatory, and dynamic (performances, exhibitions)	Making the assessment process secret, exclusive, and fixed
Giving students multiple opportunities to demonstrate their fullest mathematical ability	Restricting students to a single way of demonstrating their mathematic knowledge
Developing a shared vision of what to assess and how to do it	Developing assessment by oneself
Using assessment results to ensure that all students have the opportunity to achieve their potential	Using assessment to filter and select students out of the opportunity to learn mathematics
Aligning assessment with curriculum and instruction	Treating assessment as independent of curriculum or instruction
Basing inferences on multiple sources of evidence	Basing inferences on a single source of evidence
Viewing students as active participants in the assessment process	Viewing students as the objects of assessment
Regarding assessment as continual and recursive	Regarding assessment as sporadic and conclusive
Holding all concerned with mathematical learning accountable for assessment results	Holding only a few accountable for assessment results



## **Authentic Assessment in the Problem-Solving Classroom**

Assessment in the problem-solving classroom provides students with the opportunity to express their learning through many modalities and resources.

These are some ways in which students can be assessed in the problem-solving classroom:

**Anecdotal records**  
**Checklists**  
**Interviews/Conferences**  
**Inventories/Surveys**  
**Portfolios**  
**Rubrics**  
**Math Journals**

**Anecdotal Records** – anecdotal records are brief or simple positive notes written about the student’s interaction with the teacher, other students, the environment, and/or materials. Anecdotal records can be instrumental in capturing observations about what concepts the student understands and whether scaffolding or enrichment may be beneficial. These notes are usually informal and based on direct observation. Anecdotal notes are used to capture the richness of the learning experiences, provide written documentation in a portfolio, or guide further instruction or curriculum planning.

<http://www.ncrel.org/sdrs/areas/issues/methods/instrctn/in5lk37.htm>  
[www.fcs.utah.edu/info/cfdc/2610/anecdotal\\_records.doc](http://www.fcs.utah.edu/info/cfdc/2610/anecdotal_records.doc)

**Checklists** – checklists are a fast way to document whether students mastered a requirement for a lesson or unit. The advantages of the checklists are they are easy to develop and they clearly identify the learning expectation(s). The disadvantage is the teacher has limited feedback on the student’s method or strategy.

Possible procedures for making/using a checklist:

- Write the students’ names down the left-hand side of the paper.
- Write the expected behaviors, skills, or processes to be observed along the top of the paper.
- Write entries or note behaviors in the appropriate cells.
- Identify your key of symbols that represent the quality of the work/performance observed.

**Sample Checklist:**

(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

Week of:	Rote Counting	Counts on	Counts back	Uses 1 to 1 correspondence
April 1				
Jonathan	+ (80+)	— (inconsistent)	+	+
Thomasina	- (to 19)	—		+

Other samples can be found at <http://www.rock-hill.k12.sc.us/departments/instructionandaccountability/elementaryeducation/mathproblematictasks.aspx>

**Interviews/Conferences** – interviews and conferences are typically formal, planned conversations with students. They commonly focus on a preset of skills or topics and the questions are determined ahead of time.

Possible procedure for developing interview or conference questions:

- Plan a list of questions ahead of time. Have some questions for all students and a few for a select group of students.
- Be accommodating, and follow each student’s lead.
- Pay attention to what the students are saying. Ask for clarification if needed.
- Give the students manipulatives, in case they cannot clearly express their understanding with words.
- Reword questions, if necessary. Do not show signs of frustration.
- Keep notes for future reference.
- Avoid questions that give you right answers (to definitions or procedures). Ask questions that give you an idea of how the students are thinking.

**Sample Interview/Conference Prompts:**

Tell me more about that.  
 Can you show me another way?  
 Help me understand.  
 Why did you ...?  
 How did you know what to do next?  
 What else do you know about ...?  
 What were you thinking when you ...?

(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

**Inventories/Surveys** – inventories and surveys can give teachers information about a student’s attitude about learning math. Inventories/surveys may be given at the beginning of the year or beginning of a unit and then given other times throughout the year to determine if attitudes have changed or are different for various topics.

Possible procedure for developing inventories or surveys:

- Identify what you want to know.
- Plan a list of questions ahead of time.
- The questions maybe read to younger students or typed on a form for older students.

**Sample Inventory/Survey Prompts:**

**Indicate --- yes, almost always – sometimes – no, not really**

- I like using manipulatives to help me with my math.
- I like learning about new things in math.
- What I like best in math is ....
- I listen to directions and I follow directions

(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

**Sample Inventory/Survey Prompts:**

**Mark the face that matches your face.**

This is how I feel about ...

Math class .....	▲	▲	☹	.....
Multiplication .....	▲	▲	☹	.....
Writing about math .....	▲	▲	☹	.....
Participating in math class .....	▲	▲	☹	.....
Working with a group .....	▲	▲	☹	.....

(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

**Portfolios** – portfolios are a compilation of student work. This work should be authentic and created by the student (no commercially produced worksheet). The pieces maybe chosen by the student or chosen by the teacher; nevertheless, the students should know the reason for collecting their work. Student portfolios can also be digital portfolios, such as a blog or the Portfolio feature in Canvas.

Possible reasons for collecting student work:

- To display or praise work the students like the most or considers outstanding
- To demonstrate the students’ development and understanding of a concept
- To illustrate a representative sample of the students’ most prized work and work that shows their progress

**Sample Portfolio Entries:** The students should include an explanation with work.

Work that showed what the students have learned

Tasks that were really new and hard

Work the students did with a group

Ideas from their journals

Work that demonstrates specific concepts addressed in the current unit

**Rubrics** – a rubric is a set of criteria to evaluate an assignment and indicate the level of completion. Rubrics can be broad or detailed, and they can be holistic or analytic. Broad rubrics identify the levels of learning for any assignment or problem. The detailed rubric has some parts that are like the broad rubric, but will also include certain expectations for a particular task. A holistic rubric evaluates the student’s performance based on the whole task where one score is assigned. An analytic rubric evaluates and gives each part of the task a score. The scores are then totaled to represent the entire task.

Possible procedures for developing a rubric:

- Clear explanations of what is considered superb work and what is required as proof of learning
- A set of behavior criteria that explains the lowest level of performance
- A set of behavior criteria that is unmistakably higher than what a typical student would perform
- A description(s) of the behavior that represents the middle level of learning

### Sample Rubrics:

#### A Holistic Rubric

(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

4	Fully accomplishes the purpose of the task. Shows a good understanding and use of the main ideas of the problem. Communicates thinking clearly, using writing, calculations, diagrams and charts, or other representations.
3	Substantially accomplishes the purpose of the task. Shows a reasonable understanding and use of the main ideas of the problem. Communicates thinking fairly well, but may use only one representation.
2	Partially accomplishes the purpose of the task. Shows partial but limited grasp of the main mathematical ideas. Recorded work may be incomplete, misdirected, or not clearly presented.
1	Shows little or no progress in accomplishing the purpose of the task. Shows little understanding of the main mathematical ideas. Work is almost or completely impossible to decipher.

## A Broad Analytic Rubric

(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

Understanding the problem	0	Complete misunderstanding of the problem
	1	Part of the problem misunderstood or misinterpreted
	2	Complete understanding of the problem
Planning a solution	0	No attempt, or totally inappropriate plan
	1	Partially correct plan based on correct interpretation of part of the problem
	2	Plan that could have led to a correct solution if implemented properly
Getting an answer	0	No answer, or wrong answer based on an inappropriate plan
	1	Copying error; computational error; partial answer
	2	Correct answer and correct label for the answer

**Math Journals** – a math journal is a journal entry in which the student writes about the experience received from a specific math investigation or problem solving activity. Journal writing is a very valuable assessment technique because it provides an opportunity for students to think through and then communicate through writing what was done and what was required to solve the specific math problem. Math journaling also provides an opportunity for students to reflect and self-assess what they have learned.

Possible procedure for using a math journal:

- Should be written in at the end of the exercise
- Should be written in when introducing a new concept
- Should contain specific details regarding areas of difficulty and areas of success or to determine growth in problem solving
- Should not take more than 5-7 minutes

### Sample Math Journal Prompts:

If I missed \_\_\_\_\_ I would have to \_\_\_\_\_.

Tips I would give a friend to solve this problem are.....

Could you have found the answer by doing something different? What?

Was this hard or easy? Why?

I knew my answer was incorrect when ....

I found my mistake when ...

My answer makes sense because...

What other strategies could you use to solve this problem?

Were you frustrated with this problem? Why or why not?

What decisions had to be made when solving this problem?

Is math your favorite subject? Why or why not?

I still wonder ...

<http://math.about.com/library/weekly/aa123001a.html>

### **Creating Assessment Tasks:**

Assessment tasks should shift the focus from correct answer tasks to tasks that require an explanation. One way to do this is when the assessment asks for specific answers or skills, ask students to give explanations instead.

For example, instead of asking students the answers to  $6 + 6$  and  $6 + 7$ , ask them to write in their math journals about how knowing the answer for one problem helps with the other.

Instead of giving a routine word problem, provide the answer and ask for a justification.

Instead of having students do an entire page of adding fractions with common denominators give them the problem  $1/4 + 3/2$  and ask, “Why is a common denominator necessary?” or “Is there a strategy for getting the solution without using common denominators?” Have them use their tools to support their written explanations.

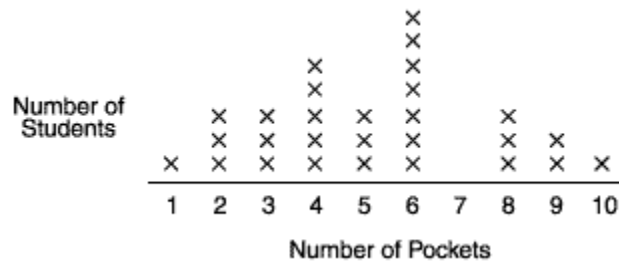
## Sample Assessment Tasks:

Logan has 34 marbles, Jennifer has 27 marbles, and Chris has 23 marbles. Write and solve as many problems as you can that use this information and represent adding two-digit numbers.

In the fraction  $\frac{3}{4}$ , how would you explain the meaning of the top number to a third-grader? How would you explain the meaning of the bottom number? Can you explain it in two different ways?

Make a triangle with one right angle and two sides of equal length. Can you make more than one triangle with this set of properties? If so, what is the relationship of the triangles to one another?

Here is a graph. What does it tell you?



At the Dollar Tree, one dollar buys two ballpoint pens. Wal-Mart sells three of the same pens for two dollars?

- Which is the better buy?
- Can you explain your answer in two different ways?

Jamal invited seven of his friends to lunch on Saturday. He thinks that each of the eight people (his seven guests and himself) will eat one and a half sandwiches. How many sandwiches should he make? Be able to explain your solution using two different strategies.

Dave says  $13 \div 4$  is  $3 \frac{1}{4}$ . Martha says it's 3 R 1. Zach says they are both wrong. He thinks it's 3.25. Why are all three of them correct? Describe a situation where Dave's answer makes the most sense. Describe one where Zach's answer is reasonable.

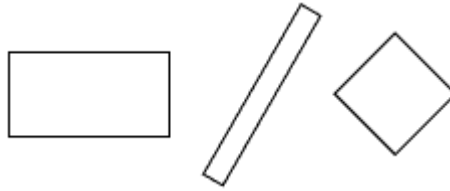
Think about the number 6 broken into 2 different amounts. Draw a picture to show a way that 6 things can be in 2 parts. Think up a story to go with your picture.

How many different ways can you make 27 cents, using pennies, nickels, dimes, or quarters? Did you find all the ways? How do you know?



Before the lesson:

Tell me everything you can about these shapes:



After the lesson: Have the students create a rubric of criteria. Ask the same question as above.

The assessment tools on the following pages will assist your grade level in developing common formative assessments. Each teacher should review his/her assessment using the Reflecting on Assessment tool on the next page. After the individual review, the grade level teachers should have collegial dialogue on how to best assess students to meet the specific learning goals.

## Template for Creating a Common Assessment

**Overview:**

What unit is this? What essential understanding is being addressed?

**Pacing:**

When in the unit will this assessment be used?

**Purpose:**

What is the purpose of this assessment?

**Content for Assessment:**

Standards being addressed:

**Content for Assessment:**

Standards being addressed:

**Content for Assessment:**

Standards being addressed:

**Anticipated Number and Types of Items on Assessment:**

___ Fill in the Blank	___ Label diagram/chart
___ Multiple Choice	___ Short Answer
___ Matching	___ Other

*Adapted from Building Common Assessments Workshop with Cassandra Erkens, Solution Tree*

## **What Makes a Quality Assessment?**

The following questions can be asked in the assessment design process to ensure high quality assessments:

- Are the concepts and skills worth mastering? (Essential learnings)
- Are assessment questions, prompts, tasks aligned to the exact learning targets?
- Will this assessment give me an accurate picture of student understanding right now?
- Do the assessment items require thought and application of knowledge? (Rigor)
- Are there multiple right answers for assessment items?
- Does the assessment appeal to varied student learning styles?
- Does this assessment lend itself to provide quality feedback to students on how they can improve?
- Have we created a rubric or performance criteria informing students about their proficiency on this assessment?

## Reflecting on Assessment (Form A)

Teacher \_\_\_\_\_

Answer the following questions about your classroom assessment you are analyzing.

1. How many questions are on the test? \_\_\_\_\_
2. Are the questions short answer, multiple choice, or more open-ended which require explanation? Are there any problem solving type questions on the test?  
\_\_\_\_\_
3. How many students in YOUR class scored 80% or above? \_\_\_\_\_
4. How many students in the grade level scored 80% or above? \_\_\_\_\_
5. Reviewing the questions on your test, can the students know the content and get the questions wrong? Can the students not know the content and get the questions right? \_\_\_\_\_
6. What patterns do you see in students' errors? (Did students compute incorrectly? Did they comprehend the questions incorrectly? Carelessness?)  
\_\_\_\_\_  
Do you and your grade level agree that the test is developmentally appropriate for your students? Why or why not?  
\_\_\_\_\_
7. Is this test teacher created, or textbook generated? (Textbook generated includes copy and pasted) \_\_\_\_\_
8. What information/knowledge did you gain **about your students** by looking more in depth at this assessment? Be specific: Do you need to re-teach any concepts? Do you need to challenge students more? Were ALL students successful in demonstrating their knowledge? What evidence does this test give you? \_\_\_\_\_
9. What information did you gain about your assessment of students? Is it authentic? Does it give them opportunities to explain their thinking? Does it apply to real-world situations? Does it give every student the opportunity to be successful? Does it communicate that you have high expectations of ALL learners?  
\_\_\_\_\_

## Reflecting on Assessment (Form B)

Teacher \_\_\_\_\_

Answer the following questions about your classroom assessment you are analyzing.

1. What specific skills or concepts are you observing/discussing with the student?  
\_\_\_\_\_
2. How did the students demonstrate their knowledge of a particular skill or concept? How did you document the students' knowledge?  
\_\_\_\_\_
3. How many students in YOUR class met 80% or more of the skills/concepts? \_\_\_\_\_
4. How many students in the grade level performed 80% or more of the skills/concepts?  
\_\_\_\_\_
5. Reviewing the questions you asked or the expected behaviors you observed, can the students know the content and not be able to demonstrate the skills? Can the students not know the content and demonstrate the skills?  
\_\_\_\_\_

6. What patterns do you see in students' errors/challenges with performing specific skills/concepts? (Did students compute incorrectly? Comprehend the questions incorrectly?)  
\_\_\_\_\_

Do you and your grade level agree that the assessment is developmentally appropriate for your students? Why or why not?  
\_\_\_\_\_

7. Is this assessment teacher created, or textbook generated? (Textbook generated includes copy and pasted) \_\_\_\_\_
8. What information/knowledge did you gain **about your students** by looking more in depth at this assessment? Be specific: Do you need to re-teach any concepts? Do you need to challenge students more? Were ALL students successful in demonstrating their knowledge? What evidence does this test give you?  
\_\_\_\_\_
9. What information did you gain about your assessment of students? Is it authentic? Does it give them opportunities to explain their thinking? Does it apply to real-world situations? Does it give every student the opportunity to be successful? Does it communicate that you have high expectations of ALL learners?  
\_\_\_\_\_

**Resources:**

Burns, M. (2007). About teaching mathematics: A K-8 resource. Third edition. Sausalito, CA: Math Solutions Publications.

National Council of Teachers of Mathematics. (1995). Assessment standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

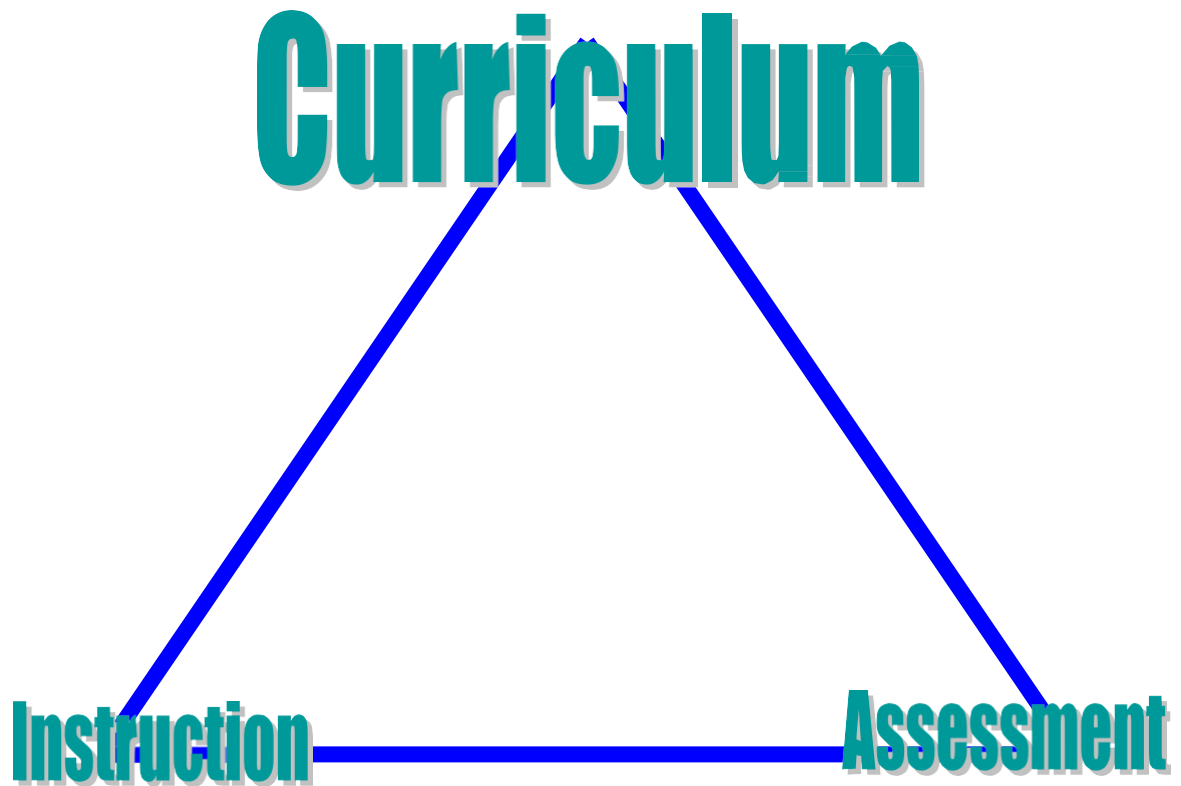
National Council of Teachers of Mathematics. (2003). Mathematics Assessment – A Practical Handbook for Grades K – 2. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2005). Mathematics Assessment – A Practical Handbook for Grades 3 – 5. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Van de Walle. (2006). Elementary and middle school mathematics: Teaching developmentally. White Plains, NY: Allyn & Bacon

# Curriculum



“The curriculum is to be thought of in terms of activity and experience rather than knowledge to be acquired and facts to be stored.” Haddow Report UK 1931

**Resources:**

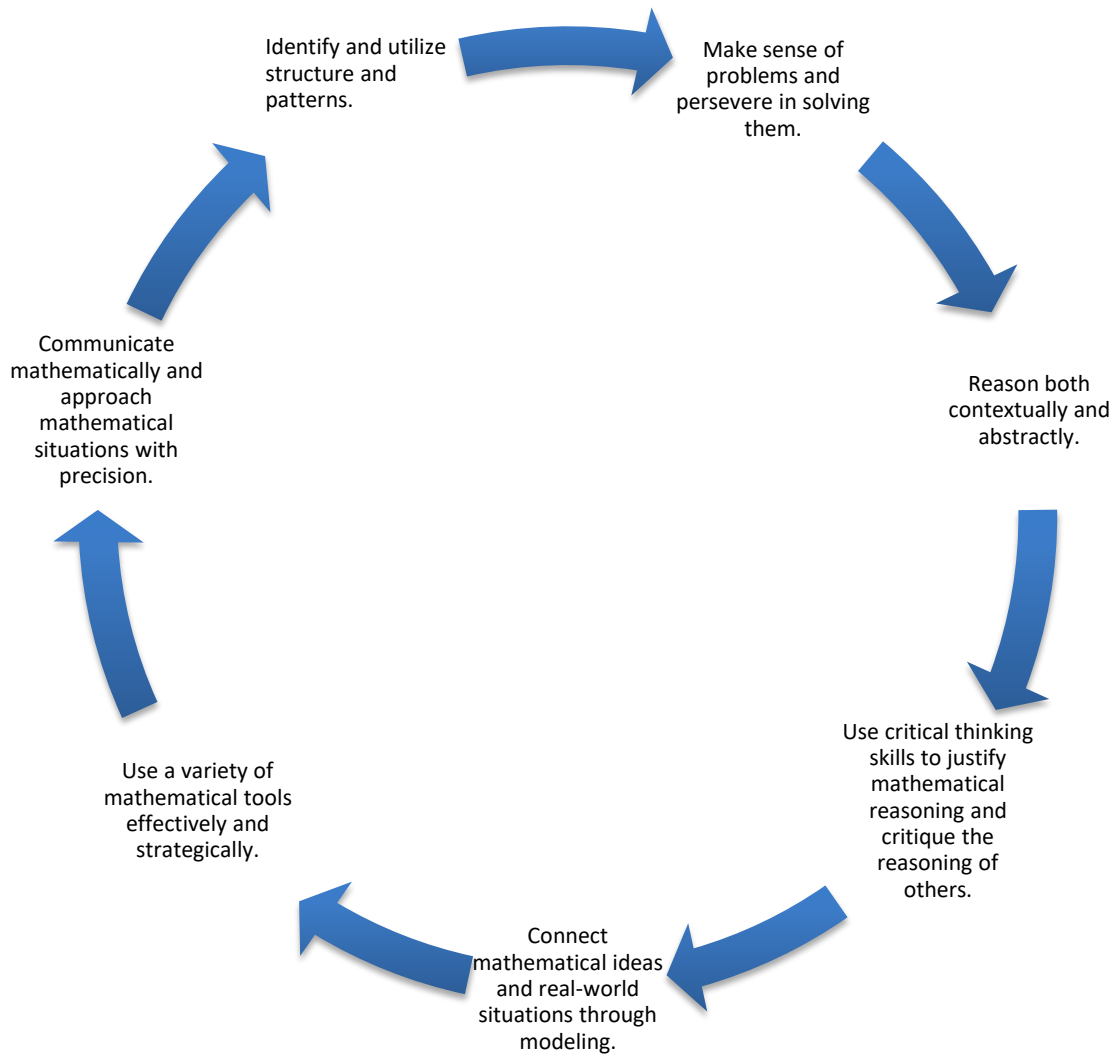
South Carolina College and Career Readiness Standards

South Carolina Support Documents

Rock Hill School Support Documents



# Mathematical Process Standards







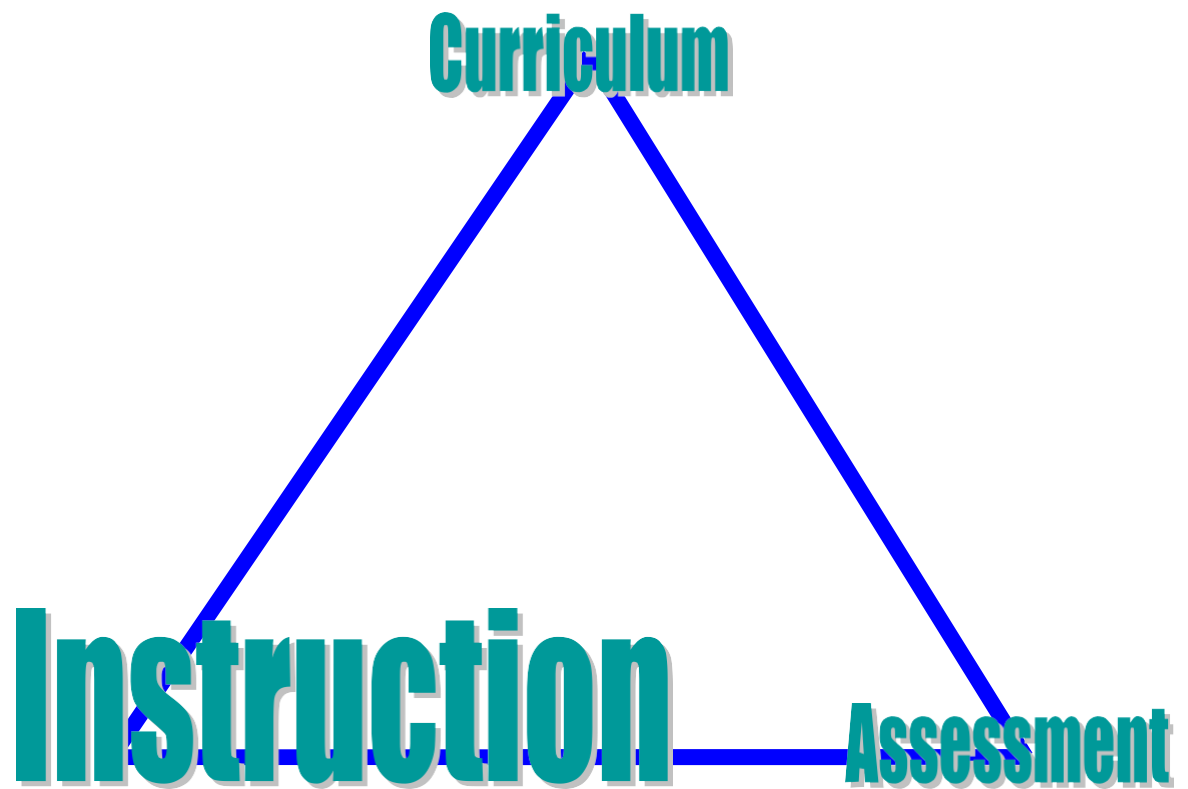
## Standards of Student Practice in Mathematics Proficiency Matrix

Students:		(I) = Initial	(IN) = Intermediate	(A) = Advanced
1a	<b>Make sense of problems</b>	<p>Explain their thought processes in solving a problem one way.</p> <p><i>(Pair –Share)</i></p>	<p>Explain their thought processes in solving a problem and representing it in several ways.</p> <p><i>(Question/Wait time)</i></p>	<p>Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways.</p> <p><i>(Grouping/Engaging)</i></p>
1b	<b>Persevere in solving them</b>	<p>Stay with a challenging problem for more than one attempt.</p> <p><i>(Question/Wait time)</i></p>	<p>Try several approaches in finding a solution, and only seek hints if stuck.</p> <p><i>(Grouping/Engaging)</i></p>	<p>Struggle with various attempts over time, and learn from previous solution attempts. <i>(Show Thinking)</i></p>
2	<b>Reason abstractly and quantitatively</b>	<p>Reason with models or pictorial representations to solve problems.</p> <p><i>(Grouping/Engaging)</i></p>	<p>Are able to translate situations into symbols for solving problems.</p> <p><i>(Grouping/Engaging)</i></p>	<p>Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations.</p> <p><i>(Encourage Reasoning)</i></p>
3a	<b>Construct viable arguments</b>	<p>Explain their thinking for the solution they found.</p> <p><i>(Show Thinking)</i></p>	<p>Explain their own thinking and thinking of others with accurate vocabulary.</p> <p><i>(Question/Wait time)</i></p>	<p>Justify and explain, with accurate language and vocabulary, why their solution is correct.</p> <p><i>(Grouping/Engaging)</i></p>
3b	<b>Critique the reasoning of others.</b>	<p>Understand and discuss other ideas and approaches.</p> <p><i>(Pair – Share)</i></p>	<p>Explain other students’ solutions and identify strengths and weaknesses of the solution.</p> <p><i>(Question/Wait time)</i></p>	<p>Compare and contrast various solution strategies and explain the reasoning of others. <i>(Grouping/Engaging)</i></p>
4	<b>Model with Mathematics</b>	<p>Use models to represent and solve a problem, and translate the solution to mathematical symbols.</p> <p><i>(Grouping/Engaging)</i></p>	<p>Use models and symbols to represent and solve a problem, and accurately explain the solution representation.</p> <p><i>(Question/Prompt)</i></p>	<p>Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem.</p> <p><i>(Show Thinking)</i></p>

5	<b>Use appropriate tools strategically</b>	Use the appropriate tool to find a solution. (Grouping/Engaging)	Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. (Grouping/Engaging)	Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution. (Show Thinking)
6	<b>Attend to precision</b>	Communicate their reasoning and solution to others. (Show Thinking)	Incorporate appropriate vocabulary and symbols when communicating with others. (Allowing Struggle)	Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. (Encourage Reasoning)
7	<b>Look for and make use of structure</b>	Look for structure within mathematics to help them solve problems efficiently (such as $2 \times 7 \times 5$ has the same value as $2 \times 5 \times 7$ , so instead of multiplying $14 \times 5$ , which is $(2 \times 7) \times 5$ , the student can mentally calculate $10 \times 7$ . (Question/Prompt)	Compose and decompose number situations and relationships through observed patterns in order to simplify solutions. (Allowing Struggle)	See complex and complicated mathematical expressions as component parts. (Encourage Reasoning)
8	<b>Look for and express regularity in repeated reasoning</b>	Look for obvious patterns, and use if/ then reasoning strategies for obvious patterns. (Grouping/Engaging)	Find and explain subtle patterns. (Allowing Struggle)	Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function. (Encourage Reasoning)

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<http://www.ascd.org/ascd-express/vol8/805-parker.aspx>

# Instruction



“I cannot teach you anything. I can only make you think.”  
Socrates

# Marilyn Burns: 10 Big Math Ideas

## Everyone's favorite math guru shares the top 10 ways you can enhance students' math learning, test scores, and skills

By Marilyn Burns | March 2005

<p><b>1. Success comes from understanding.</b></p>	<p>Set the following expectation for your students: Do only what makes sense to you. Too often, students see math as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors-when subtracting, students will subtract the smaller from the larger rather than regrouping; or when dividing, they'll omit a zero and wind up with an answer that is ten times too small. In these instances, students arrive at answers that make no sense, and they rarely know why.</p> <p>Help students understand that they should always try to make sense of what they do in math. Always encourage them to explain the purpose for what they're doing, the logic of their procedures, and the reasonableness of their solutions.</p>
<p><b>2. Have students explain their reasoning.</b></p>	<p>During math lessons, probe children's thinking when they respond. Ask: Why do you think that? Why does that make sense? Convince us. Prove it. Does anyone have a different way to think about the problem? Does anyone have another explanation?</p> <p>When children are asked to explain their thinking, they are forced to organize their ideas. They have the opportunity to develop and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their responses are incorrect. It's important, however, to ask children to explain their reasoning at all times.</p>
<p><b>3. Math class is a time for talk.</b></p>	<p>Communication is essential for learning. Having students work quietly-and by themselves-limits their learning opportunities. Interaction helps children clarify their ideas, get feedback for their thinking, and hear other points of view. Students can learn from one another as well as from their teachers.</p> <p>Make student talk a regular part of your lessons. Partner talk-sometimes called "turn and talk" or "think-pair-share"-encourages students to voice their ideas. Giving them a minute or so to talk with a neighbor also helps students get ready to contribute to a discussion. It's especially beneficial to students who are generally hesitant to share in front of the whole class.</p>
<p><b>4. Make writing a part of math learning.</b></p>	<p>Writing in math class best extends from children's talking. When partner talk, small-group interaction, or a whole-class discussion precedes a writing assignment, students have a chance to formulate their ideas before they're expected to write. Vary writing assignments. At the end of a lesson, students can write in their math journals or logs about what they learned and what questions they have. Or ask them to write about a particular math idea-"what I know about multiplication so far," or "what happens to the sums and products when adding even and odd numbers." When solving a problem, encourage students to record how they reasoned. Writing prompts on the board can help students get started writing. For example: Today I learned ..., I am still not sure about ..., I think the answer is ..., I think this because....</p>
<p><b>5. Present math activities in contexts.</b></p>	<p>Real-world contexts can give students access to otherwise abstract mathematical ideas. Contexts stimulate student interest and provide a purpose for learning. When connected to situations, mathematics comes alive. Contexts can draw on real-world examples.</p> <p>Contexts can also be created from imaginary situations, and children's books are ideal starting points for classroom math lessons. After reading Eric Carle's <i>Rooster's Off to See the World</i> (Simon &amp; Schuster, 1991), for example, ask children if they can figure out how many animals went traveling. Or ask children to follow the calculations in Judith Viorst's <i>Alexander, Who Used to Be Rich Last Sunday</i> (Simon &amp; Schuster, 1978), and figure out how Alexander spent his money.</p>
<p><b>6. Support learning with manipulatives.</b></p>	<p>Manipulative materials help make abstract mathematical ideas concrete. They give children the chance to grab onto mathematics ideas, turn them around, and view them in different ways. Manipulative materials can serve in several ways-to introduce concepts, to pose problems, and to use as tools to figure out solutions. It's important</p>

	that manipulatives are not relegated to the early grades but are also available to older students.
<b>7. Let your students push the curriculum.</b>	Avoid having the curriculum push the children. Choose depth over breadth and avoid having your math program be a mile wide and an inch deep. As David Hawkins said in <i>The Having of Wonderful Ideas</i> , by Eleanor Duckworth (Teachers College Press, 1996), "You don't want to cover a subject; you want to uncover it." There are many pressures on teachers, and the school year passes very quickly, but students' understanding is key. Explore topics that interest the students more deeply, and take the time for side investigations that can extend lessons in different directions.
<b>8. The best activities meet the needs of all students.</b>	<p>Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience. A wonderful quality of good children's books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur simultaneously at all levels. The same holds true for math. Look for activities that allow for students to seek their own level and that also lend themselves to extensions.</p> <p>For example, challenge children to find the sum of three consecutive numbers, such as <math>4 + 5 + 6</math>. Ask them to do at least five different problems and see if they can discover how the sum relates to the addends. (The sum is always the middle number tripled.) Allowing the children to select their own numbers to add is a way for students to choose problems that are appropriate for them. Even those students who don't discover the relationship will benefit from the addition practice. Invite more able students to write about why they think the sum is always three times the middle number, or to investigate the sums of four consecutive numbers.</p>
<b>9. Confusion is part of the process.</b>	Remember that confusion and partial understanding are natural to the learning process. Don't expect all children to learn everything at the same time, and don't expect all children to get the same message from every lesson. Although we want all students to be successful, it's hard to reach every student with every lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It's important that children do not feel deficient, hopeless, or excluded from learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.
<b>10. Encourage different ways of thinking.</b>	There's no one way to think about any mathematical problem. After children respond to a question (and, of course, have explained their thinking!), ask: Does anyone have a different idea? Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you'll not only learn more about individual children's thinking, but you'll also send the message that there's more than one way to look at any problem or situation. That's when the potential for delight occurs.

### About the Author

Marilyn Burns is the creator of Math Solutions, inservice workshops offered nationwide, and the author of numerous books and articles. She is author of the book *50 Problem-Solving Lessons, Grades 1-6*, distributed by Cuisenaire.



## Getting Started

Classroom organization addresses all components of instruction – **teaching strategies**, **student grouping assignments**, and **assessing** (Slavin, 1989).

The classroom begins with all students understanding the classroom expectations of respect and quality work. The **high expectations** should be modeled for the students at all grade levels. The classroom should be organized where students have DAILY access to their **math tools** and **manipulatives**. The “**toolboxes**” should be readily available and easily accessible to everyone in the classroom.

The classroom should be arranged so that the students can easily transition from independent work to group/partner assignments (and vice versa) with minimal loss of instructional time. The students should have a clear view of the strategy demonstration area. Group areas or stations/centers should be pre-determined and prearranged before students begin transitioning to them to complete assignments.

The classroom **environment** should be **kid-friendly** and **safe**. Students are encouraged to (actively) participate in class which includes verbally and physically. The students are not afraid to ask questions and to offer differing opinions. Students should be given opportunities to explore different ways/strategies for solving problems. Students’ thoughts should be validated, and their questions and concerns should be addressed. However, the teacher should redirect misconceptions through questioning and demonstrations without crushing the students’ desire for learning and trying. It is important for teachers to establish this “risk free zone” so ALL students want to and will participate in the math investigations. All students have something to contribute to the learning and growth of their teacher and peers.

### Building Your Instructional Tool Kit

To meet the diverse academic needs of the students in the class, learn as much about the students as possible. Use testing data and anecdotal data from student surveys and observations. After getting a sense of “where” the students are academically, design lessons and investigations to challenge the students.

It is recommended to:

- design the “on-grade” level lesson first; then
- adapt that lesson to meet the needs of the students below grade level and above grade level
- provide varied learning experiences over several days, the varied learning experiences include opportunities for STUDENTS to use concrete objects, visual representations, technology, etc to learn the concept or skill through problem solving situations.

The goal is to address as many of the different learning styles and modalities as possible; so that all students have equal access to the curriculum.

### **Vocabulary**

Students should be introduced to the math vocabulary through active participation in the investigations. Introducing the vocabulary out of context or as an isolated word does not give students a reference point to connect the word to. Throughout the investigation, the teacher should use the correct math vocabulary. During the discussion portion of the lesson, the students should define the significant terms related to the lesson based on the teacher's questions and examples and based on their experiences during the investigation. It is recommended that the math term is to be recorded in the math notebook or journal with a teacher definition, a definition in the students' own words, and with a picture. The students should be encouraged daily to use the correct/appropriate math vocabulary in their discussions and writings.

### **Calculators**

Calculators should be recognized as an instructional tool – similar to rulers and other manipulatives. As a result, students should have access to the calculators everyday (Van de Walle, 2000). In the primary grades (kindergarten – second), the calculator is for exploration. As the students show mastery of math facts and basic skills, the calculator should be used to reinforce the facts. In the upper elementary grades (third – fifth), the calculator should be used as another strategy to learn concepts. Students should have daily opportunities to use the calculator as another learning tool, especially in other content areas that rely on math (such as science) and for topics that require the application of basic skills (calculating the mean, perimeter, etc.). After students have shown understanding and mastery of an algorithm, the calculator should be used to check work.

### **Manipulatives**

Manipulatives should be used everyday in the math classroom. They are necessary tools, especially for our visual and kinesthetic learners. Manipulatives make many concepts seem less abstract and confusing. Manipulatives allow students the opportunity to make changes within a problem to determine patterns and to draw conclusions. Students need opportunities to experience and manipulate tools that assist them in making sense of the math.

Initially, it is recommended to:

- give the students time to explore a particular manipulative before beginning math instruction with the manipulative
- introduce a new manipulative with an old concept, especially if the students have no prior experience with the manipulative
- model two to three simple problems with the manipulative, then allow the students opportunities to use the manipulative to complete additional problems based on the same concept or skill
- reconvene with the class to discuss what the students have learned and their results from using the manipulatives

- Encourage students to think of other manipulatives that could be used to address the same concept/skill.

Successful use of manipulatives in a classroom is a combination of the teachers' high expectations and challenging and engaging lessons in a "risk-free zone" of mutual respect for learning.

**NOTE:** Not all students will be comfortable using every manipulative. Some students will "get it" or will know the answer without using the manipulatives. Those students should still be encouraged to show or explain to the teacher how they arrived at the correct solution. The ultimate goal of using manipulatives is for the students to have conceptual understanding of a concept.

For a list of grade level appropriate manipulatives, please review the list at the end of this section.

Resources:

Math Strategies You Can Count On by Char Forsten

The First Days of School by Harry K. Wong

The Differentiated Math Classroom by Miki Murray

Elementary School Mathematics Teaching Developmentally by John A. Van De Walle

Teaching Student-Centered Mathematics by John A. Van de Walle and LouAnn H. Lovin

## 10 Best Practices in Mathematics Instruction

<b>Opportunity to Learn</b>	Students' opportunity to learn mathematics is determined by the concepts and skills that are taught, the strategies used to teach the skills, and the correlation between students' current skills and the new concepts.
<b>Focus on Meaning</b>	Connecting the mathematics to other math skills and to other subject areas, providing students opportunities to develop their understanding of a concept, and acknowledging and encouraging students' different strategies to acquire meaning of concepts demonstrate teachers' abilities to focus on meaning.
<b>Learning New Concepts and Skills While Solving Problems</b>	Developing students' conceptual understanding through problem-solving does not have to be introduced after students master basic skills. The skills can be scaffold using technology.
<b>Opportunities for Both Invention and Practice</b>	Lessons should be well-balanced to provide students experiences to practice skills and opportunities to generate (invent) strategies for solving problems and developing true understanding of concepts. Strategies to develop students' invention skills include using non-routine problems, introducing a lesson involving a new skill by posing it as a problem to be solved, and allowing students to build new knowledge based on instinctive and informal procedures.
<b>Openness to Student Solution Methods and Student Interaction</b>	Teachers encourage and give students opportunities to try different solutions to solve problems. The students are also expected to share their solution methods with their peers.
<b>Small-Group Learning</b>	Small group instruction is an effective strategy for some learning goals; however, the practice is more effective when expectations are clear, there are group goals with individual accountability, and the students are taught how to work in groups and how to give and receive assistance.
<b>Whole-Class Discussion</b>	Whole-class discussion provides an opportunity for important ideas to be shared either by the students, by the teacher, or by both. Whole-class discussion is an important strategy to use after expectations (assessing - without personally attacking - others' ideas and reasoning, actively listening, and taking responsibility for the learning in the class) have been established.
<b>Number Sense</b>	Number sense is the ability of students to instinctively work with numbers using a variety of strategies and in different ways to determine accurate solutions. To help students improve their numeracy skills, teachers should discontinue teaching skills as individual topics and begin to combine the learning of skills and concepts through problem-solving and computational strategies.
<b>Concrete Materials</b>	Manipulatives should be used regularly in the math classroom to help students develop conceptual understanding of topics. The materials should be used purposefully, <b>by students</b> , in a variety of ways for multiple concepts and skills to help students understand the connections in mathematics.
<b>Student Use of Calculators</b>	Calculators should be used at all grade levels for different purposes. They are useful tools for providing instant feedback and allowing students to focus on the concept without being burdened by the computations, especially in problem-solving situations.

Cawelti, Gordon. *Handbook of Research on Improving Student Achievement*, Third Edition. Educational Research Service, copyright 2004.  
<http://jeffcoweb.jeffco.k12.co.us/isu/math/instruction/ers.htm>

# **Teaching and Learning Mathematics with Understanding**

# Differentiated Instruction for Math

## What is Differentiated Instruction?

**Differentiated instruction**, also called differentiation, is a process through which teachers enhance learning by matching student characteristics to instruction and assessment. Differentiated instruction allows all students to access the same classroom curriculum by providing entry points, learning tasks, and outcomes that are tailored to students' needs (Hall, Strangman, & Meyer, 2003). Differentiated instruction is not a single strategy, but rather an approach to instruction that incorporates a variety of strategies.

Teachers can differentiate content, process, and/or product for students (Tomlinson, 1999). Differentiation of **content** refers to a change in the material being learned by a student. For example, if the classroom objective is for all students to subtract using renaming, some of the students may learn to subtract two-digit numbers, while others may learn to subtract larger numbers in the context of word problems. Differentiation of **process** refers to the way in which a student accesses material. One student may explore a learning center, while another student collects information from the web.

Differentiation of **product** refers to the way in which a student shows what he or she has learned. For example, to demonstrate understanding of a geometric concept, one student may solve a problem set, while another builds a model.

When teachers differentiate, they do so in response to a student's readiness, interest, and/or learning profile. **Readiness** refers to the skill level and background knowledge of the child. **Interest** refers to topics that the student may want to explore or that will motivate the student. This can include interests relevant to the content area as well as outside interests of the student. Finally, a student's **learning profile** includes learning style (i.e., a visual, auditory, tactile, or kinesthetic learner), grouping preferences (i.e., individual, small group, or large group), and environmental preferences (i.e., lots of space or a quiet area to work). A teacher may differentiate based on any one of these factors or any combination of factors (Tomlinson, 1999).

Differentiation strategies support all students (gifted, varied socio-economic backgrounds, ethnic/racial groups, etc). The strategies provide all students the opportunity to learn the content (Dacey & Lynch, 2007)

## How Is it Implemented?

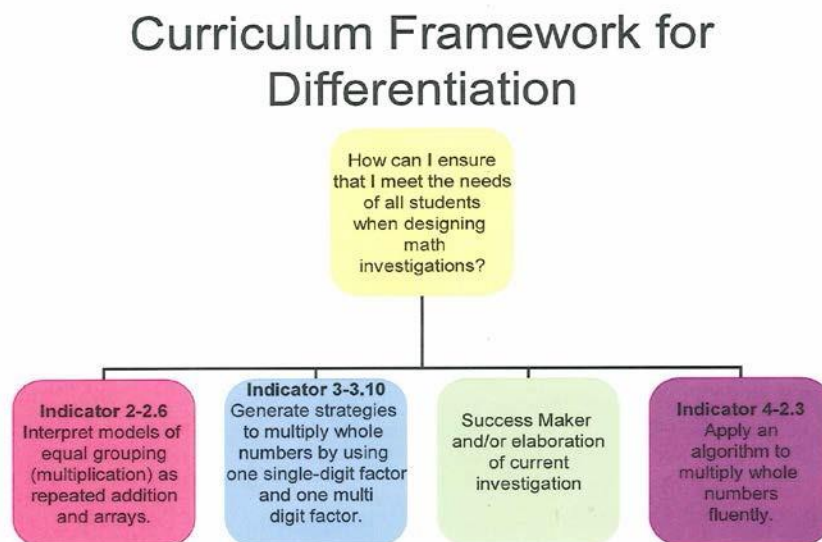
Implementation looks different for each student and each assignment. Before beginning instruction, teachers should do three things:

1. Use diagnostic assessments to **determine student readiness**. These assessments can be formal or informal. Teachers can give pre-tests, question students about their background knowledge, or use KWL charts (charts that ask students to identify what they already **Know**, what they **Want** to know, and what they have **Learned** about a topic).

2. **Determine student interest.** This can be done by using interest inventories and/or including students in the planning process. Teachers can ask students to tell them what specific interests they have in a particular topic, and then teachers can try to incorporate these interests into their lessons.
3. **Identify student learning styles and environmental preferences.** Learning styles can be measured using learning style inventories. Teachers can also get information about student learning styles by asking students how they learn best and by observing student activities. Identifying environmental preferences includes determining whether students work best in large or small groups and what environmental factors might contribute to or inhibit student learning. For example, a student might need to be free from distraction or have extra lighting while he or she works.

Teachers incorporate different instructional strategies based on the assessed needs of their students. Throughout a unit of study, teachers should assess students on a regular basis. This assessment can be formal, but is often informal and can include taking anecdotal notes on student progress, examining students' work, and asking the student questions about his or her understanding of the topic. The results of the assessment could then be used to drive further instruction.

The curricular framework, in the previous section of this guide, can assist teachers in developing lessons to meet the needs of their students. The figure below provides an example of differentiating a concept based on the students' readiness levels.



Students (in the pink box) who are having trouble generating strategies can be encouraged to make a model of equal grouping. Students (in green box) who are able to generate concrete strategies should be challenged to generate additional strategies, including pictorial strategies. Students (in purple box) who are able to generate concrete and pictorial strategies can be shown the more abstract algorithm as an additional strategy that can be used to perform the procedure efficiently...**NOTE**...they must have a solid conceptual understanding before moving into this phase.

### What Does it Look Like for Math?

Math instruction can be differentiated to allow students to work on skills appropriate to their readiness level and to explore mathematics applications. The chart below offers a variety of strategies that can be used.

<b>Strategy</b>	<b>Focus of Differentiation</b>	<b>Definition</b>	<b>Example</b>
Tiered assignments	Readiness	Tiered assignments are designed to instruct students on essential skills that are provided at different levels of complexity, abstractness, and open-endedness. The curricular content and objective(s) are the same, but the process and/or product are varied according to the student's level of readiness.	In a unit on measurement, some students are taught basic measurement skills, including using a ruler to measure the length of objects. Other students can apply measurement skills to problems involving perimeter.
Compacting	Readiness	Compacting is the process of adjusting instruction to account for prior student mastery of learning objectives. Compacting involves a three-step process: (1) assess the student to determine his/her level of knowledge on the material to be studied and determine what he/she still needs to master; (2) create plans for what the student needs to know, and excuse the student from studying what he/she already knows; and (3) create plans for freed-up time to be spent in enriched or accelerated study.	A third grade class is learning to identify the parts of fractions. Diagnostics indicate that two students already know the parts of fractions. These students are excused from completing the identifying activities, and are taught to add and subtract fractions.
Interest Centers or Interest Groups	Readiness Interest	Interest centers (usually used with younger students) and interest groups (usually used with older students) are set up so that learning experiences are directed toward a specific learner interest. Allowing students to choose a topic can be motivating to them.	Interest Centers - Centers can focus on specific math skills, such as addition, and provide activities that are high interest, such as counting jelly beans or adding the number of eyes on two aliens.  Interest Groups - Students can work in small groups to research a math topic of interest, such as how geometry applies to architecture or how math is used in art.



Strategy	Focus of Differentiation	Definition	Example
Flexible Grouping *	Readiness Interest Learning Profile	Students work as part of many different groups depending on the task and/or content. Sometimes students are placed in groups based on readiness, other times they are placed based on interest and/or learning profile. Groups can either be assigned by the teacher or chosen by the students. Students can be assigned purposefully to a group or assigned randomly. This strategy allows students to work with a wide variety of peers and keeps them from being labeled as advanced or struggling.	The teacher may assign groups based on readiness for direct instruction on algebraic concepts, and allow students to choose their own groups for projects that investigate famous mathematicians.
Learning Contracts	Readiness Learning Profile	Learning contracts begin with an agreement between the teacher and the student. The teacher specifies the necessary skills expected to be learned by the student and the required components of the assignment, while the student identifies methods for completing the tasks. This strategy (1) allows students to work at an appropriate pace; (2) can target learning styles; and (3) helps students work independently, learn planning skills, and eliminate unnecessary skill practice.	A student decides to follow a football team over a two-month period and make inferences about players' performances based on their scoring patterns and physical characteristics. The student, with the teacher's guidance, develops a plan for collecting and analyzing the data and conducting research about football. The student decides to create a PowerPoint presentation to present his or her findings to the class.
Choice Boards	Readiness Interest Learning Profile	Choice boards are organizers that contain a variety of activities. Students can choose one or several activities to complete as they learn a skill or develop a product. Choice boards can be organized so that students are required to choose options that focus on several different skills.	Students are given a choice board that contains a list of possible activities they can complete to learn about volume. For example, students can choose to complete an inquiry lesson where they measure volume using various containers, use a textbook to read about measuring volume, or watch a video in which the steps are

Strategy	Focus of Differentiation	Definition	Example
			explained. The activities are based on the following learning styles: visual, auditory, kinesthetic, and tactile. Students must complete two activities from the board and must choose these activities from two different learning styles.

\* More information about grouping strategies can be found in *Strategies to Improve Access to the General Education Curriculum*. Available at [http://www.k8accesscenter.org/training\\_resources/curricular\\_materials.asp](http://www.k8accesscenter.org/training_resources/curricular_materials.asp)

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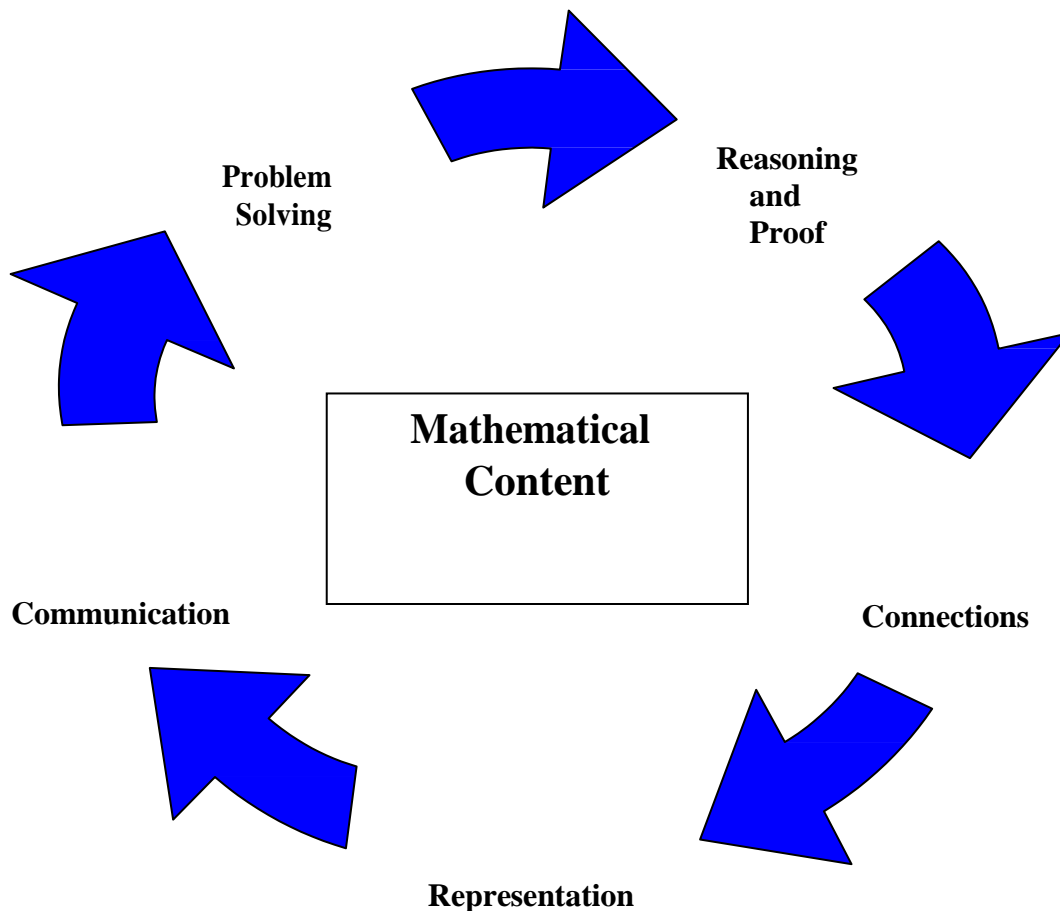
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<http://www.cast.org/ncac/index.cfm?i=2876> – This site contains an article by Tracy Hall at the National Center for Accessing the General Curriculum. The article discusses differentiation as it applies to the general education classroom.

<http://members.shaw.ca/priscillatheroux/differentiatingstrategies.html> - The Enhancing Learning with Technology site provides explanations for various differentiation strategies.

# Process Standards



The **process standards must be integrated daily into mathematics instruction**. They should never be taught as a separate stand alone unit. Students should learn the mathematical content standards *through...not in addition to...*the process standards. Therefore, math content should be the result of well-designed problem situations that require students to reason mathematically, communicate with one another, use connections and representations to support their efforts, and justify their reasoning through valid arguments.

## Process Standard: Problem Solving

**Instructional programs from pre-kindergarten through grade 12 should enable all students to--**

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

**NCTM (2000)**

### What does it mean to teach math through problem solving?

The National Council of Teachers of Mathematics states in the *Principles and Standards for School Mathematics* (2000) that understanding must be the goal for all of the mathematics we teach. It is important to realize that understanding cannot be taught directly. Teaching through problem solving is an approach that allows students to learn mathematics with understanding as they use various solution methods to solve problems. “*Most, if not all, important mathematics concepts and procedures can best be taught through problem solving* (Van de Walle, page 11, 2006). Students should have frequent opportunities to formulate, grapple with, and solve problems that require a significant amount of effort and should be encouraged to reflect on their thinking (NCTM, 2000, pg. 51).

The emphasis in problem solving has shifted from teaching problem solving to teaching *through* problem solving. The focus is on teaching mathematical topics through problem-solving contexts and inquiry-oriented environments which are characterized by the teacher “helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying (Lester et al., 1994). This is in opposition to the approach in which the skills in the chapter are taught directly and then the students are expected to apply those skills as they complete the few word problems at the end of the chapter. Teaching through problem solving reverses the process so that math skills and concepts become the valuable outcome of the problem-solving experience as opposed to a prerequisite for solving the problem.

### Why should I teach math through problem solving?

There are many reasons for making the curriculum switch to teaching through problem solving. Although it may be more difficult initially, the reward is that your students will learn the math you teach with understanding and will see that math is a subject that does make sense. Other benefits include:

- Problem solving focuses students’ attention on ideas and sense-making. This means they must reflect on the math; therefore, new ideas are more likely to be integrated with existing ones. This focus improves understanding.

- Problem solving helps students realize that they are capable of doing mathematics. When students are given problems to solve and know that you expect a solution, you are saying to them “I believe you can do this.” Self-esteem is enhanced every time they solve a problem.
- Problem-solving provides ongoing assessment data. When teachers listen to their students discuss ideas and defend solutions, a steady stream of valuable assessment information can be attained.
- Problem-solving tasks are easier to differentiate. Since students are allowed to choose strategies they want to use, they can use methods to solve the problem that make sense to them. Their understanding is also increased as they hear others share their solution strategies.
- Problem-solving naturally engages students in all five of the process standards: problem solving, reasoning and proof, communication, connections and representation.
- Problem-solving helps students learn how to construct their own methods to solve problems and apply these methods to new problem-solving situations. This allows them to solve a variety of problems without having to memorize different procedures for each new problem (Hiebert, 1996).
- Problem-solving is engaging. For this reason, students are more likely to actively participate resulting in fewer discipline problems (Van de Walle, 2000).

### **What are the Characteristics of a Problem-Solving Approach**

- Interactions between students/students and teacher/students
- Mathematical dialogue and consensus between students
- Teachers providing just enough information to establish background/intent of the problem, and students clarifying, interpreting, and attempting to construct one or more solution processes (Cobb et al., 1991)
- Teachers accepting right/wrong answers in a non-evaluative way
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994)
- Teachers knowing when it is appropriate to intervene, and when to step back and let students make their own way
- **When using a problem-solving approach to teaching mathematics, the remaining four process standards of communication, connections, representation, and reasoning and proof can be easily integrated into classroom instruction**

### **How do I plan problem-based lessons?**

A problem-based approach should be used in the classroom every day. In classrooms where both traditional skill-based teaching and teaching through problems are used, children become confused about when they are supposed to use their own strategies for figuring out a problem and when they are supposed to use the “teacher’s approach” (Mokros, Russell, and Economopoulos, 1995).

**Step 1** – Decide on the math indicator that you plan to teach.

**Step 2:** Think about your students. What do they already know and understand about this topic? Is there some background information they need before being able to solve the problem?

**Step 3:** Decide on a task. Keep it simple! Good tasks need not be elaborate. Often a simple story problem is all that is necessary as long as the solution involves children doing the intended mathematics.

**Step 4:** Predict what will happen. Think about the strategies that the students might use, but be prepared to see the students using strategies that you never considered.

**Step 5:** Plan the “mini lesson.” This is the part of the lesson where you introduce the problem to be solved and review any background concepts or math vocabulary that is important for solving the problem. It is important to refrain from “teaching” the students how to go about solving the problem.

**Step 6:** Plan the small group portion of the lesson. What will they do? How can you facilitate their efforts without telling them “how” to solve the problem? What kinds of questions might you ask?

**Step 7:** Plan the after portion of the lesson. This is the part of the lesson in which students share with the class their findings and the strategies they used to solve the problem. This is a very important part of the lesson and should never be omitted (Van de Walle, 2000).

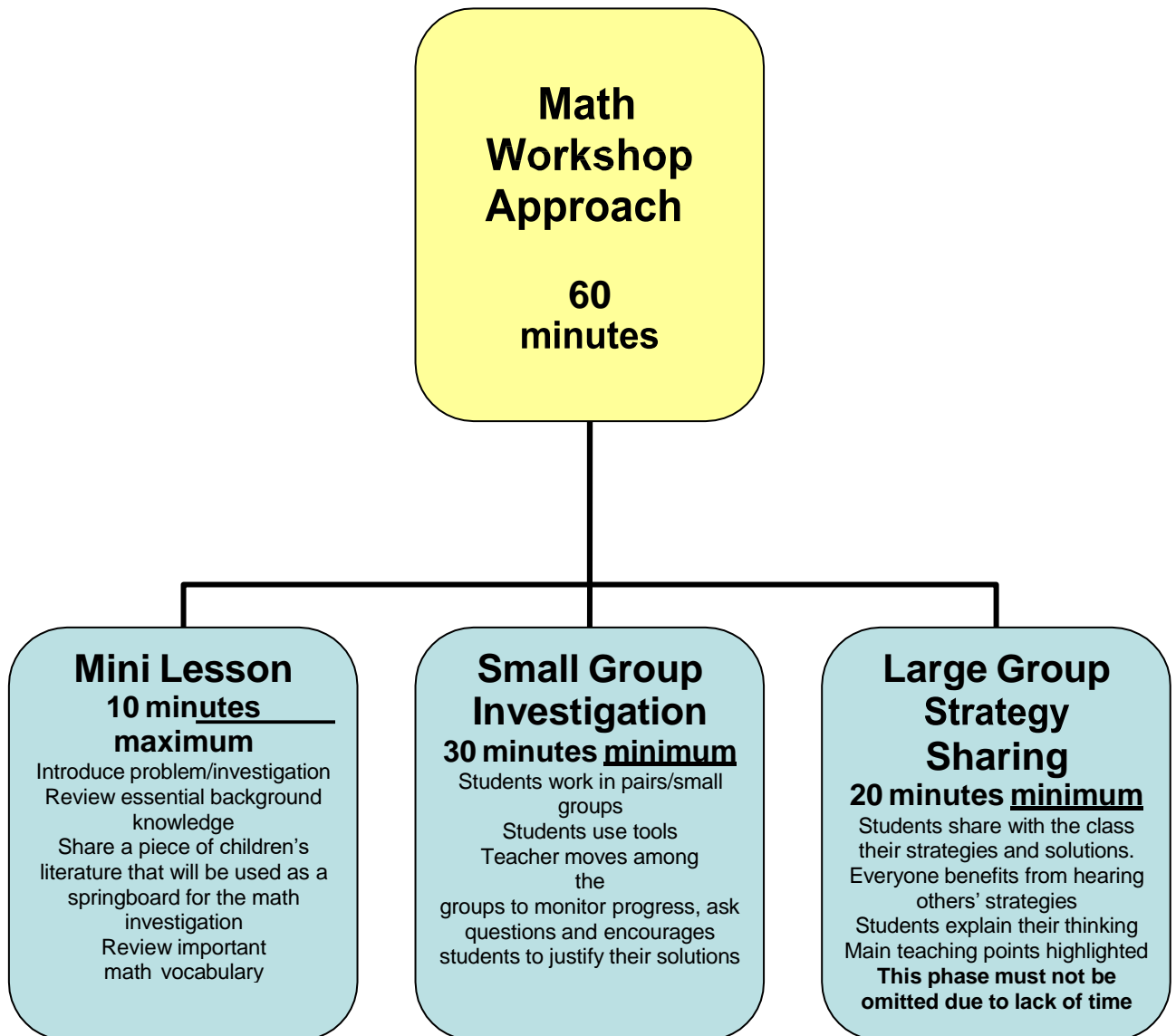
## Math Workshop

Problem-solving lessons should be implemented using the Math Workshop approach. Math Workshop begins with a mini lesson, progresses to small group work, and ends with a whole group discussion. **A MINIMUM OF 60 MINUTES SHOULD BE DEVOTED TO MATH WORKSHOP.**

**Mini Lesson:** 10 – 15 minutes **maximum**

**Small Group Work** – 30 minutes **minimum**

**Large Group Strategy Sharing:** 20 – 30 minutes **minimum**



## What does a problem-solving lesson look like?

### Mini Lesson

During the mini lesson, the teacher presents or reviews concepts and poses the problem to be investigated. Allow time for children to ask about the meaning of the task or ask questions about the task. Explain the task to them if necessary but do not *tell* students *how* to solve it. Review any background math concepts or math vocabulary. Do not directly teach the new concepts. These will emerge through the process. An investigation could be introduced by posing a problem to solve or sharing a piece of children's literature that leads to a problem for the students to solve.

### Small Group Work

Students should work in pairs or small groups to solve the problem. This allows them to communicate their ideas to others. A variety of tools (concrete materials, technology, etc) should be available for them to use. The teacher's role is to monitor their progress and encourage them to look for multiple solution strategies. They should be encouraged to use whatever strategies make sense to them in solving the problem. If a group is stuck, facilitate by asking questions that can help them get started. The teacher should also differentiate instruction by providing different types or levels of small group investigations or problems to solve.

### Whole class Discussion

This is an important phase. Ask the groups to share their solutions, strategies and explain their thinking. Students should be asked to explain how they arrived at their solutions whether those solutions are correct or incorrect. Many times, when students have an incorrect solution, the process of talking about how they arrived at the solution will cause them to realize their error without having to be told by the teacher. An additional bonus in having the students share their strategies is that others see strategies that they had not considered and thus gain a wider repertoire of problem-solving strategies.

In most cases, students will make the main teaching points for you during the whole class discussion since they have "constructed" this new knowledge by being actively engaged in "using" the mathematics to solve the problem. Nevertheless, it is important to summarize the discussion for everyone, emphasizing and explaining key points. Make sure you relate the new math concepts back to the task the children have been working on so that the discussion remains meaningful.

**Small- and large-group discussion should make-up the majority of the lesson time. Therefore, there will be a much greater percentage of student talk than teacher talk.**



## **Grouping Strategies for Problem Solving**

It is important to consider the ways that we group students for problem-solving investigations. How can we make sure that all students remain interested, challenged, and supported? Here are some ways of grouping that have proven successful:

- Group students strategically by ability levels. This will provide the group with a range of math knowledge for problem solving. A good time to use this grouping pattern is when you believe that students, through their work together, can raise their level of performance.
- Randomly select students to work together for short-term projects. Invite students to create a group by lining up according to size and then count off for grouping. Another way to group is to form groups based on attributes such as those who are wearing shoes that tie, those who are wearing Velcro closings, people with brown eyes, people with blue eye, etc. You can also challenge students to make up their next way to group.
- Grouping by interest is another successful grouping strategy. Students are often motivated by particular areas of interest. When you put those students together, they often perform beyond your expectations. They may be more willing to persevere with problem solving.
- Invite students to help you develop a list of grouping ideas to use throughout the year. Developing many different grouping practices helps students understand that all members of the classroom can work with one another.

Taking the time to build these values and routines in the classroom will pay off. The classroom will be perceived as a safe place for taking the necessary risks for learning, as well as for performing optimally during assessment time (Kallick & Brewer, 1997)

## **Sample Math Workshop Lesson Plan**

<b>Lesson Title</b>
<b>SUGGESTED MATERIALS:</b>
<b>PROCESS INDICATORS ADDRESSED:</b>
<b>CONTENT INDICATORS ADDRESSED:</b>
<b>MINI LESSON</b>
<b>SMALL GROUP WORK</b>  <b>Plans for Differentiation:</b>
<b>WHOLE GROUP DISCUSSION</b>

## **Problem-Solving Steps**

Teaching students this sequence of steps will lay a strong foundation for their future problem-solving success.

### **Get Ready to Solve the Problem**

1. Read the problem first.
2. Underline, circle, or highlight the important facts.
3. What are you supposed to find out?
4. Create a plan to solve the problem.
5. Are there any “tricky” parts to the problem?
6. Is there “extra” information that is not essential in solving the problem?
7. What math vocabulary words are in the problem?
8. Which math strategies will you use?
9. Do you need manipulatives or other math tools to help you?

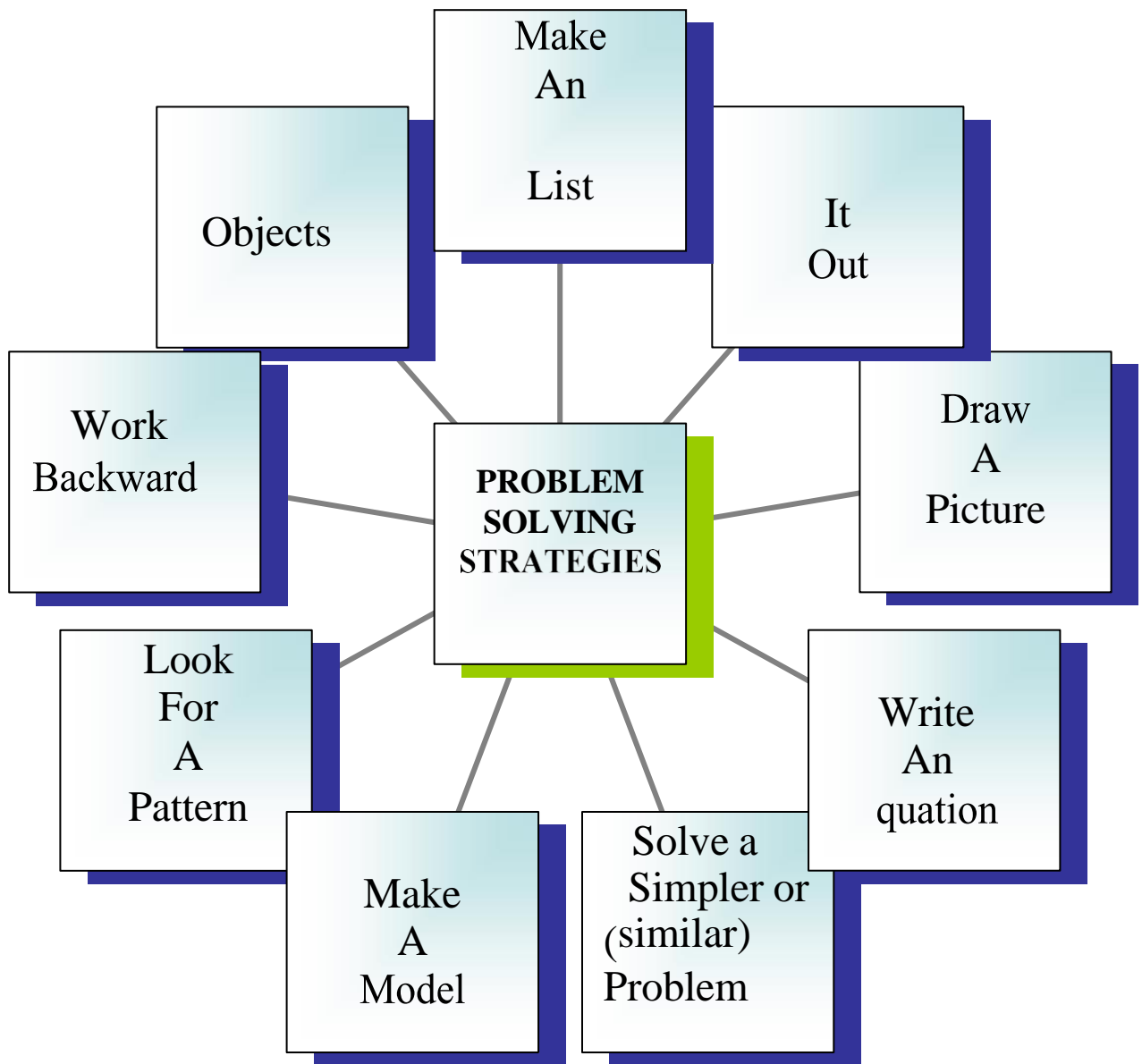
### **Solve the Problem**

1. Work together to implement the chosen strategies.
2. Use manipulatives and/or other tools (pictures, writing, talking) to facilitate the problem solving process.
3. Monitor progress regularly. Are the chosen strategies working? Do we need to try a different strategy?
4. Look for patterns that will help you solve the problem.
5. Once you have solved the problem, look at your solution and determine if it is reasonable.
6. Solve the problem using a different strategy to verify your solutions.
7. Be prepared to defend your strategies and solutions to your peers.

Remind students that these steps do not always have to be followed in this order all the time. After reading the problem and creating a plan, students may realize that the plan or strategy that they created is not the best one for that particular problem or that they misunderstood what the problem was asking. Self-correcting is also a strategy that great problem-solvers use. [http://www.homeschoolmath.net/teaching/problem\\_solving.php](http://www.homeschoolmath.net/teaching/problem_solving.php)

Model, monitor, and assess by becoming a part of the problem-solving process with students.

**Glencoe/McGraw-Hill**, 2001, *Glencoe MATHEMATICS Applications and Connections, Course 1, 2 & 3*. McGraw-Hill Companies, Inc., 8787 Orion Place, Columbus, Ohio 43240.



Adapted from *About Teaching Mathematics* by Marilyn Burns

## What kinds of problem-solving tasks should we use to teach math concepts?

- **The problem-solving task must begin at the student’s current level of understanding.** They should have the appropriate ideas to solve the problem and yet still find it challenging and interesting.
- **The task must require students to reflect and communicate about the mathematics embedded in the problem.** Reflection and communication are the processes through which understanding develops. The tasks provide the context in which students can reflect on and communicate about mathematics (Hiebert, 1996).
- **The task must allow students the opportunity to use tools.** Tools are learning supports and do not only include physical materials such as manipulatives. Other tools that should be encouraged are skills that have been previously acquired (background knowledge), written symbols, pictures, and verbal language.
- **The problem-solving task should leave behind important mathematical residue** (Hiebert, 1996). The “residue” that results should be new strategies for solving problems and a deeper understanding of the math concepts that were embedded in the task. Solving the problem must require the use of mathematical ideas and must be based on sound and significant mathematics (NCTM, 2000).
- **The task should be based on the knowledge of the range of ways that diverse students learn mathematics.** Teachers must display sensitivity to, and draw on, students' diverse background experiences and dispositions when designing tasks for students.
- **The task should be engaging.** The problem must be of interest to the students so that they have a desire to solve the problem.
- **The problem-solving task must help develop students’ understandings of the math standards.** It is critical that teachers have a standards-based focus when designing mathematical tasks. The goal is for the students to develop a solid understanding of the math indicator(s) through the process of solving problems.
- **The problem-solving task must stimulate students to make connections, reason mathematically, create mathematical representations, and promote communication about mathematics.** These process standards are easily integrated into instruction with a problem-based focus.

## How to Create Good Problem-Solving Tasks

A good problem can be used as the basis for an entire lesson. There are many resource books that contain good problems for students to solve, but it is also possible to make up your own good questions. There are two helpful approaches that can be used in creating good problems for students to solve.

### Method 1: Working Backward

Step 1: Identify a topic (indicator).

Step 2: Think of a closed question and write down the answer. Step

3: Make up a question that includes (or addresses) the answer.

For example:

Step 1: The indicator to be taught is finding the mean of a set of data.

Step 2: The closed question might be *The children in the Williams family are aged 3, 8, 9, 10, and 15. What is the mean of their ages? The answer is 9.*

Step 3: The good question could be *There are five children in a family. Their average age is 9. How old might the children be?*

### Method 2: Adapting a Standard Question

Step 1: Identify a topic.

Step 2: Think of a standard question.

Step 3: Adapt it to make a good question.

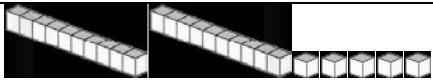
For example:

Step 1: The topic for tomorrow is measuring length using nonstandard units.

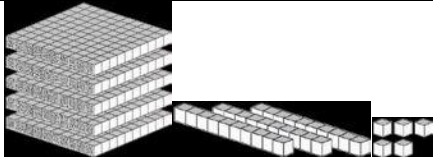
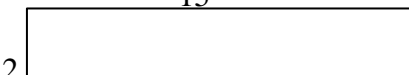
Step 2: A typical exercise might be *What is the length of your table measured in orange Cuisenaire Rods?*

Step 3: The good question could be *Can you find objects in the classroom that are 10 Cuisenaire Rods long?*

## Two Types of Problem-Solving – Grades K-2

Routine Problems	Non-Routine Problems
 <p>What number do these blocks represent?</p> <p>a) 125 b) 25 c) 15 d) 51</p>	<p>Using base 10 blocks, how many different ways can you show the number 25? Which way uses the most number of blocks and which way uses the fewest number of blocks?</p>
<p>Lisa has 2 dimes, 4 nickels and 5 pennies in her pocket. How much money does she have in all?</p>	<p>Lisa has coins in her purse that have a total value of 45 cents. What coins might she have? What coins might she have if she has exactly 11 coins in her pocket?</p>
<p>I bought a pencil for 45 cents. I gave the cashier 50 cents. How much change did I get?</p>	<p>I bought something and got 5 cents change. How much did it cost and how much money did I give to pay for it?</p>
<p>There are 19 children in Mrs. Johnson's class. Is this an even number or an odd number?</p>	<p>When the children in a class each got a partner, there was one child left over. How many children might be in the class?</p>
<p>What is the length of your desk?</p>	<p>How many objects can you find that are longer than 1 foot but shorter than 2 feet in length?</p>
<p>John ate 4 apples. Janice ate 8 apples. How many apples did they eat altogether?</p>	<p>John and Janice ate 12 apples. How many apples might they each have eaten?</p>
<p>There are 6 tables in the classroom. Four students sit at each table. How many students are in the classroom?</p>	<p>A class has 24 students. They sit at tables in the classroom. Each table has the same amount of students. How many tables might there be in the classroom and how many students might sit at each table?</p>
<p>Jimmy has 14 marbles. Logan has 10 marbles. How many marbles do they have altogether?</p>	<p>Jimmy and Logan are playing marbles. They have 24 marbles between the two of them. Jimmy has more marbles than Logan. How many marbles might each of them have?</p>
<p>A basketball player scored 5 points in her first games and 4 points in her second game. How many points did she score in both games?</p>	<p>A basketball player scored 9 points in two games. What might her scores in each of the games be?</p>
<p><math>6 + 5 + 2 = \underline{\quad}</math></p>	<p><math>? + ? + ? = 13</math>. What might the missing numbers be?</p>

## Two Types of Problem-Solving – Grades 3-5

Routine Problems	Non-Routine Problems
I bought lunch and received 3 quarters, 2 dimes, and 1 nickel in change. How much change did I receive in all?	I gave change of \$1.00 using quarters, dimes, and nickels. How might the change have looked?
Round 348 to the nearest tens.	A number has been rounded to 350. What might the number be?
Write >, <, or = 1.5 ___ 2 1.9 ___ 2 1 ___ 1.3	I am thinking of some decimal numbers between 1 and 2. What might they be? Give at least 15 answers.
 <p>What number do these blocks represent?</p> <p>e) 125 f) 25 g) 15 h) 525</p>	Using base 10 blocks, how many different ways can you show the number 535? Which way uses the most number of blocks and which way uses the fewest number of blocks?
Which of the following numbers are divisible by 3? 6, 28, 18, 12	What do you know and what can you find out about the multiples of 3.....3, 6, 9, 12, 15, 18, 21...?
$34 + 36 + 38 = \underline{\quad}$	Three consecutive even numbers add up to a number between 100 and 200. What might the numbers be?
$23 + 9 = \underline{\quad}$	Make up some different ways to add 9 to 23 in your head. In how many ways can you do it?
$100 \div 20 = \underline{\quad}$	The answer to a division question is 5. What might the question be?
Eighty-four children were divided into 4 equal teams. How many children were in each team?	Eighty-four children in four grades are arranged into teams with the same number on each team. How many teams are there and how many children might there be on each team?
Is 370 divisible by ten?	What could you add to 361 to make it divisible by 10?
Find the area of the rectangle. <div style="text-align: center;">15</div> 	I am thinking of a shape with an area of thirty square tiles. What might the shape look like?



I built a garden in the shape of a rectangle. The length of my garden was 75 feet and the width was 25 feet. Find the perimeter and area of my garden.	I want to make a vegetable garden in the shape of a rectangle. I have 200 feet of fence for my garden. What might the area of the garden be?
I left for school at 7:15 and arrived at 8:00. How long did it take me to get to school?	I left home and arrived at school forty-five minutes later. When might I have left home and when might I have arrived at school?
The football game started at 7:30 PM and lasted two hours and fifteen minutes. What time did the game end?	A football game lasted two hours and fifteen minutes. What might be suitable starting and finishing times?
The length of the paperclip is about ___ inch (s). The width of your desk is about ____ feet.	How many things can you find that are 1 inch long? One foot long?
Circle the correct answer. It will snow in Rock Hill in November. Likely or Unlikely	My older sister was talking to Dad and asked him a question. His reply was, "It is more <i>likely</i> than <i>unlikely</i> ." What might the question be?

Sullivan, P. & Lilburn, P. (2002). *Good questions for math teaching: Why ask them and what to ask (K-6)*. California: Math Solutions Publications.

## PROCESS STANDARD PROMPTS

<p><b><u>Connections Prompts</u></b></p> <p>Does this remind you of anything we've done before?          Can someone think of a time when you've needed to...(measure, add, subtract, etc.)?          How is this idea related to...(addition, subtraction, multiplication, etc.)?          When might a (scientist, chef, doctor, architect, etc.) need to use what we're learning today?          How is what we're learning today important to you in your everyday life?          Would it help you to try to solve a simpler problem?          Can you use what we learned about addition to help you solve this new problem?          This reminds me of the problem we solved last week.          What patterns did we discover when solving that problem and how can you use those same patterns to help you solve this new problem?</p>	<p><b><u>Communication Prompts</u></b></p> <p><b><u>Oral</u></b></p> <p>What strategy might you use to solve the problem?          Talk with your group to come up with a plan to solve the problem.          Talk about what tools you might need to help you solve the problem.          Do you see a pattern that might help?          Who can explain what Rahimme said using different words?          Did anyone think about the problem in a different way?          Does anyone have any questions they want to ask Jose about his solution?          What do you notice about?          What do you find interesting?          I wonder what would happen if...?          Would anyone like to add to what JaNita just said?          Please explain that in a different way.          Will you say a little more about that?          Do you agree with what Yoshi just said?</p> <p><b><u>Written</u></b></p> <p>Write about the strategy that you used to solve the problem.          Write about how you know the answer is correct.          Write about what you discovered during this investigation.          Write about what you are confused about.          Write down any new questions you have.</p>
<p><b><u>Reasoning and Proof Prompts</u></b></p> <p>Explain why you think your answer is reasonable.          Explain why your answer makes sense?          Do you agree with Susie's explanation?          How did you get your answer?          Tell me how you thought about that.          Can you solve the problem in another way?          How can you be sure that you've found all the solutions?          Why does your solution work?          Is Kaia's idea very different from yours?          Does that seem right to you, Lisa? How can you convince yourself.          How is Karen's strategy the same as/different from Brandy's strategy?          Do you think your strategy will always work?          What discoveries did you make?          Did you notice any patterns?</p>	<p><b><u>Representation Prompts</u></b></p> <p>Can you illustrate what you're saying using your Base 10 blocks (or whatever manipulative being used)?          How can you organize your results?          Can you display your data in a graph or chart?          Can you organize your results in a T-chart (function table)?          Show me your strategy using a different manipulative.          Show me how you arrived at your solution.          Can you draw a picture to show your findings?          Describe the strategy that you used to find the answer by writing it down in your math journal.          Show me several different ways that you can present your findings to the class.</p>

## PROCESS STANDARD: PROBLEM SOLVING

### Characteristics of a Problem-Solving Approach

Continuous interaction/discussion between students/students and teacher/students.  
 Hands-on active learning  
 Students using manipulatives and other tools to find solutions.  
 Teachers providing just enough information to establish background/intent of problem but not “telling” students how to solve the problem.  
 Teachers guiding, coaching, asking insightful questions and sharing in the process of solving the problem.  
 Teachers knowing when to intervene, and when to step back.  
 Students solving open-ended problems.  
 Students reasoning, communicating, making connections, and representing mathematical ideas.

### How to Plan Problem-Based Lessons

**Step 1** – Decide on the math indicator.  
**Step 2:** Think about your students. What do they already know and understand about this topic? Is there some background information they need before being able to solve the problem?  
**Step 3:** Decide on a task. Keep it simple! Good tasks need not be elaborate. Often a simple story problem is all that is necessary as long as the solution involves children in the intended mathematics.  
**Step 4:** Predict what will happen. Think about the strategies that the students might use, but be prepared to see the students using strategies that you never considered.  
**Step 5:** Plan the “mini lesson.” This is the part of the lesson where you introduce the problem to be solved and review any background concepts or math vocabulary that is important for solving the problem. It is important to refrain from “teaching” the students how to go about solving the problem.  
**Step 6:** Plan the small group portion of the lesson. What will they do? How can you facilitate their efforts without telling them “how” to solve the problem? How will you differentiate? What kinds of questions might you ask?  
**Step 7:** Plan the after portion of the lesson. This is the part of the lesson in which students share with the class their findings and the strategies they used to solve the problem. This is a very important part of the lesson and should never be omitted.

### How to Create Good Tasks

A good problem can be used as the basis for an entire lesson. There are two helpful approaches that can be used in creating good problems for students to solve.  
**Method 1: Working Backward**  
**Step 1:** Identify a topic (indicator).  
**Step 2:** Think of a closed question and write down the answer.  
**Step 3:** Make up a question that includes (or addresses) the answer.  
 For example:  
**Step 1:** The indicator to be taught is finding the mean of a set of data.  
**Step 2:** The closed question might be *The children in the Williams family are aged 3, 8, 9, 10, and 15. What is the mean of their ages?* The answer is 9.  
**Step 3:** The good question could be *There are five children in a family. Their average age is 9. How old might the children be?*  
**Method 2: Adapting a Standard Question**  
**Step 1:** Identify a topic.  
**Step 2:** Think of a standard question.  
**Step 3:** Adapt it to make a good question.  
 For example:  
**Step 1:** The topic for tomorrow is measuring length using nonstandard units.  
**Step 2:** A typical exercise might be *What is the length of your table measured in orange Cuisenaire Rods?*  
**Step 3:** The good question could be *Can you find objects in the classroom that are 10 Cuisenaire Rods long?*

### Problem Solving Steps (not always linear)

1. Read the problem.
2. Circle/highlight the important facts.
3. What are you supposed to find out?
4. Create a plan to solve the problem.
5. Are there any “tricky” parts to the problem?
6. Which math strategies/tools will you use?
7. Work together to implement the strategies.
8. Use tools (pictures, writing, manipulatives, talking).
9. Monitor your progress. Are the strategies working?
10. Look for patterns that will help you solve the problem.
11. Look at your solution and decide if it is reasonable.
12. Solve the problem using a different strategy to verify your solutions.
13. Be prepared to defend your strategies/solutions.

## Process Standard: Communication

**Instructional programs from prekindergarten through grade 12 should enable all students to--**

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

**NCTM (2000)**

### Talking to Learn Mathematics

The process standard of communication should include oral language as well as written language. This section focuses on how “talking mathematics” is an important vehicle for helping children make sense of math and develop true mathematical understanding.

#### **How do conversations help students develop strong mathematical ideas?**

- When students talk about the mathematics they are doing, they are able to share and compare their methods and solution strategies thus gaining a wide repertoire of useful strategies.
- Math conversations help students learn new, possibly more effective, approaches by hearing others.
- Conversations allow students to examine their own concepts and ideas in light of others’ questions and/or counter assertions.
- Conversations help clarify confusion as students comment on each other’s methods and ask each other questions.
- Talking mathematics allows students to articulate and refine their ideas.
- Discussing ideas and strategies with classmates results in the creation of new mathematical ideas and theories.
- As students say things out loud, they can “hear” their errors.
- Talking supports the “construction” of new mathematical understanding.

#### **What does a math conversation look like?**

- Students sharing observations, strategies and personal stories connected to the math experience.
- Students seeking clarification by asking questions.
- Students speculating and posing new questions.
- Students brainstorming, estimating and thinking hypothetically.

- Because students are “thinking out loud” the talk is characterized by “rough-draft” talk - hesitant, pauses, false starts, trial and error, etc.
- Talk is more student-directed.
- There is a greater percentage of student talk than teacher talk. The teacher trusts that students are capable of coming up with new ideas and making connections; therefore, he/she doesn’t do all of the thinking for the students.
- There is a constant revision of ideas
- Kids listening to one another and valuing others’ comments.
- Students justifying their strategies and solutions.
- Students constantly searching for patterns to use in analyzing mathematical situations.
- Students agreeing, disagreeing, challenging one another’s ideas.
- Students clarifying and revising thoughts, strategies, ideas, and solutions.
- Students building off one another’s ideas.

**Teacher’s role:**

- Provide worthwhile mathematical problems; ones that challenge each students’ thinking.
- Ask open-ended questions.
- Model patterns of discourse, i.e. making an argument, asking questions (see below for examples).
- Respect and value all contributions.
- Incorporate small group work regularly.
- Model active listening.
- Have students discuss in small groups before whole group convenes.
- Allow more wait time; allow time for everyone to think.
- Share your own thinking as you work through problems.
- Think aloud yourself.
- Pose questions and problems.
- Allow informal language; don’t expect polished speech.
- Be a co-participant in the conversation.
- Use mathematical vocabulary but don’t demand that students use the vocabulary when sharing.
- Struggle for solutions with the students.
- Participate without dominating. Know when to intervene.
- Be curious and wonder about mathematics.
- Don’t interpret everything the students say and repeat to the group (if every comment is filtered through the teacher, there is little chance that the students will develop a conversation among themselves).
- Do math yourself; participate in a mathematical culture.

## Example of a Mathematical Conversation

In this scenario, a second-grade teacher posed the problem of figuring out her weight based on the weight of her four-month-old baby. It is clear to see how the students' understanding of how to solve the problem is clarified as a result of hearing and building on one another's ideas and theories. The teacher's role was one of allowing the students to share, questioning and responding when necessary, accepting all responses correct or incorrect, and finally challenging the students to justify their strategies and solutions by working in groups and using tools to help them make sense of the problem.

**Teacher:** *Can you believe my baby weighs 12 pounds now? He's growing so fast!  
And I'm exactly 10 times as heavy as my baby.*

**Brandy:** *(excitedly) Can I do that problem?*

**Shaquasia:** *I figured it out!*

**Tonya:** *All ya gotta do is add 12 plus 10.*

**William:** *Twelve times ten.*

**Larry:** *Yeah, twelve times ten.*

**Teacher:** *Twelve times ten?*

**Demetrius:** *120.*

**Teacher:** *Will you talk why you would say twelve times ten and how you figured 120?*

**Demetrius:** *Because I know 10 times 10 is – um – 10 times 10 is a hundred and I added 12 and that's – um – a hundred times twelve equals twenty.*

**William:** *I like...took 12...took 10, I mean, and said 10, 20, 30, 40, 50, 60, 70, 80, 90, 100...then I went 110...and 120...and I came up with one hundred and twenty...cause I added ten...twelve times.*

**Demetrius:** *That's what I did!*

**Megan:** *I figured out twelve times ten and then I like...I know twelve times ten....and then I did...and then I figured out I can do twelve....ten times...I mean...ten...twelve times...and then I said 10, 20, 30, 40, 50, 60, 70, 80, 90, 100...and then I said...um...I need twelve...so I counted two more and I said that equals 120...so you weigh 120 pounds.*

**Teacher:** *Okay, Larry how did you do it?*

**Larry:** *I figured out...I figured out 20...and then I figured a one and then I put the one in front of the 20 and I got 120. That's how I got it.*

**Teacher:** *Will you explain that again?*

**Larry:** *Well, somehow I got 20 and then I put a one...and then put the one in front.*

**Teacher:** *Explain the 20. How did you get that?*

**Larry:** *I have 12...and then I got 20...uh... (puzzled expression).*

**Demetrius:** *That's how I got 120. I knew ten times ten was 100...that's 100...and I needed 2 more so I added ten times two....and that equals 20.*

**Teacher:** *Okay, we have some great ideas and strategies here. What I want you to do now is to go back to your tables and work with your groups to prove to me that I weigh 120 pounds. You can use manipulatives, drawings, or equations to convince me that your solution is accurate. I need to be able to see that you understand the strategy that you use.*

### Questions to Encourage Math Talk

These questions and prompts not only encourage students to talk mathematically, but they are instrumental in getting students to reason mathematically and prove their assertions, make connections, and use representations as they work together to solve problems.

- How might you start solving the problem?
- What problem solving strategy might you use?
- Do you agree with Susie's explanation?
- Explain why you think your answer is reasonable.
- Who can explain what Rahime said using different words?
- Did anyone think about the problem in a different way?
- Does anyone have any questions they want to ask Jose about his solution?
- What do you notice about...?
- What do you find interesting?
- So... let's see...I wonder what would happen if...?
- Would anyone like to add to what JaNita just said?
- Would you explain that in a different way?
- Will someone say what John just said in a different way?
- Will you explain that again so that everyone can hear?
- Will you say a little more about that?
- Do you think there are cases where that wouldn't work?
- Is Kaia's idea very different from your idea?
- Does that seem right to you, Lisa? How could you convince yourself?
- Does this remind you of any other mathematical investigations you've done?
- Would it help you to try to solve a simpler problem?
- What can you tell me about...?
- Do you agree with what Yoshi said?



- How did you work it out?
- Does that answer make sense?
- Talk about why your answer makes sense.
- What strategy did you use?
- How is Joe's strategy different/the same as John's strategy?
- Do you see any patterns?
- Could you do it another way?
- Can you convince each other that you have found all of the possibilities?

**Resources:**

Corwin, R. (1996). *Talking mathematics: Supporting children's voices*. Portsmouth, NH: Heinemann.

Frailey, K. (2002). *Talking to learn: The potential of exploratory conversations in helping children learn mathematics*. Unpublished Doctoral Dissertation.

## Process Standard: Communication

### Writing to Learn Mathematics

Writing in math class supports learning because it requires students to organize, clarify, and reflect on their ideas – all useful processes for making sense of mathematics. In addition, writing can be useful for assessment, providing insight into students' understandings and misconceptions about the content they are studying.

Writing in math class isn't meant to produce a polished product, but rather to provide a way for students to reflect on their own learning and to explore, extend, and cement their ideas about the math they are learning. Teachers should pay attention to what the students write, not how they write it (Burns, 2007). It is important to make sure students understand that they are writing to support their learning and not to create a perfect piece.

It is helpful for students to keep a math journal for their writing. Listed below are some suggestions for incorporating writing in math instruction:

- Write about the strategy or strategies that you used to solve the problem.
- Write about how you know the answer that your group came up with is the correct answer.
- Write about what you learned today during math workshop.
- Write about what you are unsure about or confused by? Write about any new questions you have. Did solving this problem make you wonder about anything?
- Write about what was easy for you and what was difficult for you?
- Draw a picture to show how you solved the problem.
- Write about the strategy that you used and convince me that it was the best strategy for you.
- Write a story about what (addition, subtraction, division, fractions, patterns, etc.) means to you. Why is it important? When do you use it?
- Write about how (addition and subtraction, multiplication and division, fractions and decimals) are alike and different.
- Write about two shapes that we have been learning about. How are they alike and how are they different?
- Write about how you would explain to a Martian how to measure the perimeter of the classroom. Use this idea for any new math content being taught.
- Write a letter to the principal explaining to him/her what it means to be a good problem solver.
- Write a class book modeled after a piece of children's literature incorporating a mathematical concept. For example, after reading *The Doorbell Rang* and figuring out how many cookies each child should get, students can write their own division stories using new characters, setting, and context.

**Students do not have to write about every math problem/activity they do in class. Incorporate writing in math once or twice a week to help students ponder new mathematical ideas and reflect on their new mathematical learning. Don't forget to include these pieces as valuable assessment information!**

## **Process Standard: Reasoning and Proof**

Reasoning is central to making sense of and learning mathematics with understanding.

**Instructional programs from prekindergarten through grade 12 should enable all students to--**

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

### **What does reasoning and proof look like in the classroom?**

- Making discoveries or drawing conclusions as a result of thinking, explaining or justifying an idea.
- Forming conclusions, inferences, judgments.
- Being expected to provide justifications and explanations. This holds children accountable for the assertions they make and the solutions they offer.
- Explaining how and why they solved the problem in the way that they did.
- Finding and using patterns to analyze mathematical situations. Recognizing patterns is the key to the understanding of mathematical concepts.
- Defending ideas, strategies, and solutions, correct and incorrect.
- Making conjectures and supporting them by gathering evidence and building valid mathematical arguments.
- Examining, exploring, thinking about, and discussion a variety of mathematical possibilities.
- Developed through consistent use in many contexts.

### **Questions teachers can use to encourage reasoning and proof in the classroom:**

- How did you get your answer?
- Tell me how you thought about that.
- Can you solve the problem in another way?
- Why does your solution work?
- Do you think that strategy will always work?
- What discoveries did you make?
- Did you notice any patterns?

**Through the use of reasoning, students see that math makes sense (NCTM, 2000).**

## Process Standard: Connections

**Instructional programs from prekindergarten through grade 12 should enable all students to--**

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

NCTM (2000)

### **What does it mean to make connections and why is it important?**

- Making connections involves a rich interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests.
- Students should be encouraged to connect mathematical concepts to their daily lives.
- Connections should be explored and capitalized on in helping students make sense of the mathematics being examined.
- Students should connect existing knowledge and background experiences to make sense of new mathematical ideas.
- Connections should be woven into daily practice.
- Teachers need to help students be conscious and aware of the connections they make.

### **Questions teachers can use to encourage students to make connections:**

- Does this remind you of anything we've done before?
- Can someone think of a time when you've needed to...(measure, add, subtract, etc.)?
- How is this idea related to...(addition, subtraction, multiplication, etc.)?
- When might a scientist need to use what we're learning today?
- How is this important to you in your everyday lives?
- Can you use what we learned about addition to help you solve this new problem?
- This reminds me of the problem we solved last week. What patterns did we discover when solving that problem and how can you use that same pattern to help you solve this new problem?

## Process Standard: Representation

**Instructional programs from prekindergarten through grade 12 should enable all students to--**

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

NCTM (2000)

### **What is Representation and why is it important?**

- Representations are necessary to students' understanding of mathematical concepts and relationships.
- Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others.
- Representations allow students to recognize connections among related concepts and apply mathematics to realistic problems.
- Students should represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional.
- Students should also learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.
- Representations include models, manipulatives, drawings, pictures, equations, diagrams, tables, charts, graphs, symbols, mental images, words, and ideas.

### **Questions to encourage representation:**

















- Can you illustrate what you're saying using your Base 10 blocks (or whatever manipulative being used)?
- How can you organize your results?
- Can you display your data in a graph?
- Can you organize your results in a T-chart?
- Will you show me your strategy using a different manipulative?
- Will you show me how you arrived at your solution?
- Will you draw a picture to show your findings?
- Describe the strategy that you used to find the answer by writing it down in your math journal.
- Show me several different ways that you can present your findings to the class.

## Number Sense


Number sense is a construct that relates to having an intuitive feel for number size and combinations as well as the ability to flexibly work with numbers in problem situations in order to make sound decisions and reasonable judgments.

Over 90 percent of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation, or a calculator.

Cawelti, Gordon. *Handbook of Research on Improving Student Achievement*, Third Edition. Educational Research Service, copyright 2004.

-   Help students develop a deeper sense of cardinality; give them counting activities with concrete objects. The students should progress to creating sets of counters that matches a set on a card.
-   Address “more, same, less” relationships, students should be asked the following questions equally during activities - “which set is more” and then asked “which set is less”. Students should also be given opportunities to label sets (concretely and pictorially) with less, same and more cards.
-   Teach numeral writing and recognition with similar numbers together so that the students can identify their similarities and differences. The calculator is a good tool for numeral recognition where student can find and press the correct number (especially numbers they can relate to-- ages, number of brothers and sisters, number of windows in classroom, etc). (**Zaner Bloser**)
-   Practice frequent short drills (using movement and in rhythm) of counting on and backwards to improve oral counting. Using the calculator is also helpful because the students get to see the numbers as they say them with the beat.
-   Utilize concrete objects when counting on and counting back. Hide some objects under a cup or piece of paper. The students identify how many are hidden and then begin counting on to determine the total represented.
-   Continue building number sense by giving students opportunities to learn and understand relationships between numbers using patterns. Expose the students to common number patterns (such as dice or dominos) and ask the students to make the patterns on construction paper. Introduce different patterns for the same number as the students begin to learn the patterns.
-   Assist students with “one and two more/less” relationship. Ask them to find the number that is one less (more) or two more (less) than the number represented on the dot plate or domino. Students can also take turns reading/stating the resulting number sentence. The calculator can also be used to review the relationship of one or two more and less.
-   Allow students to practice showing the relationship of numbers to 5 and 10, use a ten frame (see teacher resource section of the guide) and ask the students to represent the number in the frame. Students can also practice more and less relationships using the frames. The calculator can also be used to help students understand the relationship between numbers and 5 or 10 by pressing  $\phi.95$  or 10  $-\therefore$ .

- 👉📄 Focus on a particular number throughout the investigation to help students develop part-part-whole relationships. Students use different materials and formats to create the number. It is important that students say or “read” the parts aloud and/or draw or write them down on some form of recording sheet.
- 👉📄 Introduce missing-part relationships by focusing on a particular number as well. A portion of the designated amount of materials is hidden (under a cup or piece of paper), and the students determine the hidden amount.
- 👉📄 Extend the four number relationships (visual/spatial relationships, more/less, anchors/benchmarks of 5 and 10, and part-part-whole) to numbers between 10 and 20. It is recommended to continue to use concrete objects and visual representations to show the relationships as was done with numbers up to 10.
- 👉📄 Associate the numbers with images when addressing doubles ( $6 + 6 = 12$ ). The students should draw pictures or make posters for each double. The calculator can be used to assist students with identifying doubles. The other relationships (more/less, part-part-whole, anchors with 5 and 10) should also be integrated into the study of doubles.
- 👉📄 Connect numbers with objects/situations that students can relate to, especially for estimation. Teachers are encouraged to use the following prompts to help young children begin to understand estimation:
  - More or less than \_\_\_\_? Will it be more or less than 10 footprints?
  - Closer to \_\_\_ or to \_\_\_? Will the apple weigh closer to 10 cubes or closer 30 cubes?
  - Less than \_\_\_, between \_ and \_\_\_, or more than \_\_\_\_? Are there less than 20, between 20 and 50, or more than 50 cubes in the Unifix bar?
  - About \_\_\_\_? Use one of these numbers: 5, 10, 15, 20, 25, 30, 35, 40, ... About how many footprints?
- 👉📄 Other activities for relating numbers to the students’ world include:
  - Write a number on the board for the students. Include a unit (dollars, hours, cars, meters, minutes, etc) and ask the students to state what they think of when you say the number with the unit. Change the units and ask the students for their thoughts again. During another time keep the unit the same and change the number.
  - Pick any number, large or small, and a unit with which the students are familiar. Then make up a series of questions to determine if it is reasonable. *Could the teacher be 15 feet tall? Could your living room be 15 feet wide? Can a man jump 15 feet high?*
  - Pick any number (such as seven) and have groups of children find ways to tell about that number.
- 👉📄 Graph situations that students can connect to, for example their favorite color. Once a graph is made, it is very important to take a few minutes to ask as many number questions as is appropriate for the graph and to encourage student to make up questions about the graph, as well. The graphs focus attention on counts of realistic things, an important connection. Equally important, graphs clearly exhibit comparisons between and among numbers that are rarely made when only one number or quantity is considered at a time.

 Develop “one more than/one less than” relationships to larger numbers using the base-ten-frames (or a similar tool). This activity can help the students develop their mental math abilities. Part-part-whole concepts using tools as the ten frames can also help students develop their mental math abilities. The students extend the strategies learned for single digit numbers to double digit numbers.



## Strategies for Helping Children Master the Basic Facts

*All children are able to master the basic facts – including children with learning disabilities. Children simply need to construct efficient mental tools that will help them*  
(Van de Walle, 94, 2006).

### Overview of a strategy-based approach to teaching the basic facts

An efficient strategy is one that can be done mentally and quickly. Counting is not efficient. *If drill is undertaken when counting is the only strategy available, all you get is faster counting* (Van de Walle, 95, 2006).

**Make Strategies Explicit in the Classroom** – Provide opportunities for students to discover and develop strategies as they solve story/word problems or as they investigate a category of facts you present. When a student suggests a new strategy make sure everyone else in the room understands how it is used. Don't be tempted to just tell them the strategy to use. Instead, continue to discuss strategies invented by the class and plan lessons that encourage them to invent and *practice* strategy use. Have students discuss ways that *they* can use to think of facts easily. Create a poster of strategies that students develop. Have the students create names for the strategies that make sense to them.

**Drill Established Strategies** – Once children are comfortable using a strategy mentally, drill that *particular* strategy. For example, after they have practiced using the “Make 10” strategy, provide drill by giving them the opportunity to use the “Make 10” strategy only. Games and other activities can be used to drill the use of the strategy. When drilling a strategy, students should not only provide the answer to the math fact, but they should also explain the strategy they used.

**Practice Strategy Selection or Strategy Retrieval** – After children have worked on two or three strategies, provide drill that allows them to select which strategy would be appropriate to solve certain facts.

**Children should NEVER be drilled on the basic facts apart from a focus on the strategy used. Doing this focuses on memorization as opposed to a strategic approach based on number relationships and sense-making.**

There are 100 basic arithmetic facts, zero through nine. That can be reduced by half if students understand the commutative property. Still, that is a lot for students to memorize by rote. Below are some effective strategies that students can learn that will facilitate more successful retrieval of the basic facts. Try to avoid teaching these strategies directly. Providing well-planned opportunities for students to explore and discuss the number relationships evident in the facts will go a long way in ensuring that students will understand the strategies and use them instead of the non-efficient method of counting.

## Strategies for Addition Facts

### One-More-Than and Two-More-Than Facts

Students can find sums like  $5 + 1$  and  $6 + 2$  by counting on. With practice, they will begin to be able to do this mentally without having to count up 1 or 2. This strategy allows them to check off 36 of the math facts to be learned.

**Facts with Zero** – Nineteen facts have zero as one of the addends. Word problems involving zero will help students see that not all answers to addition problems are bigger. Soon they will realize that the sum is always the other number.  $8 + 0 = 8$ ,  $0 + 4 = 4$

**Doubles** – There are 10 doubles facts from 0 to 9. A good strategy is to have students draw pictures they can use to remember the doubles. Some suggestions that teachers have used include: 3 is the bug double ( $3 + 3 = 6$  legs); 4 is the spider double ( $4 + 4 = 8$  legs); 5 is the hand double ( $5 + 5 = 10$  fingers); 6 is the egg carton double ( $6 + 6 = 12$  eggs); 7 is the calendar double ( $7 + 7 = 14$  days); 8 is the crayon box double ( $8 + 8 = 16$  crayons); 9 is the eighteen-wheeler double ( $9 + 9 = 18$  wheels). Post pictures of these in the classroom. Students will begin to develop mental images of these and will most likely remember them. Challenge students to come up with their own examples, as well.

**Near-Doubles** – These are also called the “doubles-plus-one” facts. There are 18 of these. The strategy is to double the smaller number and add 1. These should be taught after students have an understanding of the doubles facts. To introduce this strategy, you can write ten near-doubles facts on the board and allow them to solve the problems and discuss in groups their ideas for “good” methods to use. Some students may double the smaller addend and add 1, while others may double the larger addend and subtract 1. If no one uses a near double strategy, write the corresponding doubles fact and ask them to consider how they could use that to help.

**Make-Ten Facts** – These facts all have at least one addend of 8 or 9. One strategy for solving these facts is to build onto the 8 or 9 up to 10 and then add on the rest. For  $4 + 8$ , start with 8, then 2 more makes 10, and that leaves 2 more for 12. An activity to help with this is to give students two ten-frames, have them model each addend, and then decide on the easiest way to show the total. They should see that moving counters into the frame showing either 8 or 9 to fill that one up is a sensible choice – then they would just add the ones remaining to 10. Students need plenty of time to investigate with ten

frames. They also need to be able to explain what they are doing as they use the ten frames to practice the make-ten facts.

**Doubles Plus Two** – The preceding strategies cover all but 12 facts – 6 if you consider the commutative property. Of those 6, 3 of them can be remembered using the doubles plus two strategy. These are  $3 + 5$ ,  $4 + 6$ , and  $5 + 7$ . Students can double the smaller number and add 2. Students may also discover that you can take 1 from the larger addend and give it to the smaller. With this idea,  $4 + 6$  could be transformed into  $5 + 5$ . The strategy involves doubling the number *in between*.

**Make-Ten Extended** – Three of the 6 remaining facts have 7 as an addend. Through the use of ten-frames, students can build onto 7 up to ten and then add the rest; therefore,  $7 + 4$  could be thought of as  $10 + 1 = 11$ . Again, ten-frames are critical in helping students discover this strategy.

## Strategies for Subtraction Facts

**Subtraction as Think-Addition** – This strategy encourages students to think “What goes with this part to make the total?” This think-addition strategy makes use of the known addition facts. For example, when given  $9 - 4$ , children should be able to think spontaneously, “Four and *what* makes nine?” Typically, students rely on holding up 9 fingers and putting down 4. Or, they might count up from 4 or back from nine, using their fingers. Counting in this way is not an efficient strategy. Help students understand this strategy by using word problems that sound like addition but have a missing addend. *Logan had 5 Webkinz, Gran gave him some more. Then he had 12 Webkinz. How many Webkinz did Gran give Logan?* Students must have mastery of addition facts to be successful with this strategy. All of the facts can be learned using think-addition; however, there are several other strategies that students can use. These are more sophisticated strategies and should not be required of all students.

**Build up through 10** – This includes all facts where the part or the subtracted number is either 8 or 9.

Example  $14 - 8$

Start with 8

How much to 10? (2)

How much more to 14? (4)

So 14 minus 8 is (6).

**Back Down through 10** – Take  $15 - 6$ , start with the total of 15 and take off 5, that takes you down to 10, then take off 1 more to get to 9. For  $14 - 6$ , start with the total of 14, take off 4 to get to 10, then take off 2 more to get 8.

## Strategies for Multiplication Facts

**Doubles** – Facts that have 2 as a factor are the same as the addition doubles and should already be known by students who know their addition facts. Students should see that  $2 \times 7$  or  $7 \times 2$  can be thought of as  $7 + 7$ .

**Fives Facts** – Facts with 5 as the first or second factor are fairly easy for children to remember because they should be familiar with counting by 5's.

**Zeros and Ones** – Thirty-six facts have at least one factor that is either 0 or 1. These facts seem easy to adults, but children sometimes get confused as to why  $6 + 0$  stays the same, but  $6 \times 0$  is always zero and  $1 + 4$  is a one-more idea and  $1 \times 4$  stays the same. It is helpful to use story problems to help students develop the concepts behind these facts. Simply telling students that any number multiplied by zero is zero is not enough to develop a true understanding of that concept.

**Nifty Nines** – Looking at patterns in the nines facts can make the nines fairly easy to learn. There are 2 patterns that students can discover. The first is that the tens digit of the product is always one less than the “other” factor (the one other than 9). For example,  $4 \times 9$  is going to be thirty-something (3\_\_) because 3 is one less than 4. The second pattern is that the sum of the two digits in the product is always 9. These two ideas can be used together to get any nine fact quickly. For  $7 \times 9$ , 1 less than 7 is 6, 6 and 3 make 9, so the answer is 63. Children are not likely to “invent” this strategy so teachers should write the nines table on the board and encourage students to find as many patterns as they can. They can also look at the 9's row and column on a multiplication chart and discuss the patterns they see. Since the conceptual basis for this strategy will not be readily apparent to the students, they should be given the opportunity to see that the rules work because of an interesting pattern that occurs in our number system. **Teaching the students how to use this strategy will be confusing to the students unless they are encouraged to discover the patterns themselves.**

**Helping Facts** - Seventy-five of the 100 multiplication facts are covered by the 4 strategies above. That leaves 25 to learn, and it's actually 15 because 20 of them consist of 10 pairs of turnarounds.

$3 \times 3$	$4 \times 4$	$6 \times 6$	$7 \times 7$	$8 \times 8$
$3 \times 4$	$4 \times 6$	$6 \times 7$	$7 \times 8$	
$3 \times 6$	$4 \times 7$	$6 \times 8$		
$3 \times 7$	$4 \times 8$			
$3 \times 8$				

These can be learned by relating each to an already know fact or “helping” fact. For example  $3 \times 8$  is connected to  $2 \times 8$  (double 8 and 8 more). The  $6 \times 7$  fact can be related to either  $5 \times 7$  (5 sevens and 7 more) or  $3 \times 7$  (double  $3 \times 7$ ). It is important that students know the helping fact as well as have the ability to do the mental addition. There are 4 models for developing understanding of helping facts.

**Double and double again approach** – This approach is applicable to all facts with 4 as one of the factors. For  $4 \times 6$  and  $6 \times 4$ , students can double 6 to get 12 and double that to get 24. As the factors get larger ( $4 \times 8$ , double 8 to get 16, then double 16 to get 32), students should practice mental math methods ( $15 + 15$  is 30,  $16 + 16$  is 2 more, or 32). Adding  $16 + 16$  on paper defeats the purpose.

**Double and one more set approach** – This approach works with facts that have one factor of 3. For example,  $3 \times 7$  and  $7 \times 3$  can be thought of as double 7 is 14 and one more 7 is 21.

**Half then double approach** – If either factor is even, this approach can be used. Select the even factor and cut it in half. If the smaller fact is known, then that product can be doubled to get the new fact. For  $6 \times 7$ , half of 6 is 3 and  $3 \times 7$  is 21, then double 21 to get 42.

**Use a “close” fact and add one more set approach**– Some students find it easy to go to a close fact and then add one or more sets. For example, to solve  $6 \times 7$ , if students think of it as 6 sevens, they know 5 sevens is close,  $5 \times 7$  is 35 and one more set of sevens is 42.

Note: The helping fact strategies may seem hard to follow, but it remains a more effective approach for learning the 15 remaining facts than traditional rote memorization. Students can understand these strategies if given ample opportunities to explore and discover patterns and relationships within and among the facts. Again, story problems are helpful for developing understanding of these harder facts.

## Strategies for Division Facts

**Think-Multiplication** - Mastery of multiplication facts and connections between multiplication and division are the key elements of division fact mastery. For example, for  $36 \div 9$ , students should think, “nine times what is thirty-six?” If students know their multiplication facts well,  $42 \div 6$  becomes closely tied to  $6 \times 7$  and  $24 \div 6$  becomes closely tied to  $6 \times 4$ , etc. Word problems continue to be a key method for creating this connection.

**Near facts** – Divisions that do not come out evenly are more common in real situations than divisions without remainders. A useful strategy for determining  $60 \div 8$ , most people run through some of the multiplication facts in their heads – 8 times 6 (too low),  $8 \times 7$  (close),  $8 \times 8$  (too high) so it must be 7. That is 56 and 4 more. This process can and should be drilled.

The above information was adapted from *Teaching Student-Centered Mathematics* by John A. Van de Walle

## What about timed tests?

*Teachers who use timed tests believe that the tests help children learn the basic facts. This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning (Burns, 2007, pg. 192).*

### Using timed tests to help students learn the math facts:

- Does not measure children's understanding.
- Focuses on memorization, not on appropriate strategy use. When students are under pressure to complete a list of facts in a short amount of time, they will not focus on choosing and using the strategies they have learned.
- Doesn't ensure that students will be able to use the facts in problem-solving situations.
- Conveys that memorizing is what mathematics is all about, not thinking and reasoning to figure out answers.
- Has been contributed to the development of math anxiety.

**Note:** Sending home math fact tables so that parents can help their children memorize them has been a long-standing approach in the teaching of basic math facts. It is perfectly fine to have students practice their math facts at home, but the emphasis must remain on strategy use. Educating parents about the strategies that students are learning cannot be overemphasized. A homework assignment might include a list of math facts for students to complete, but should also include an area for students to describe the strategy they used to solve the fact.

# Manipulatives

Manipulatives should be used everyday in the math classroom. They are necessary tools, especially for our visual and kinesthetic learners. Manipulatives make many concepts seem less abstract and confusing. Manipulatives allow students the opportunity to make changes within a problem to determine patterns and to draw conclusions. Students need opportunities to experience and manipulate tools that assist them in making sense of the math.

To supply the elementary mathematics classroom, the following manipulatives are recommended:

## Grades K-2

Base 10 sets  
Basic Balance/Scales  
Bears or other counters  
Calculators  
Cuisenaire Rods  
Dice  
Geared Clocks  
Geoboards  
Hundreds Charts  
Inchworm Rulers Set  
Jumbo Foam Dice  
Linking Cubes  
Measurement Devices/Containers (for length, volume, capacity, and equivalencies)  
Money Collection  
Pattern Blocks  
Plastic Chips or other counters  
Rulers  
Square Tiles  
Thermometers  
Two-color bean counters  
Unifix Cubes  
Wooden Cubes  
Wooden Geometric Solids

## Grades 3-5

Calculators  
Cuisenaire Rods  
Decimal Squares (4-5)  
Dice  
Fraction Circles Set  
Fraction Squares Set  
Fraction Tower Set  
Geoboards  
Geometric solids (wooden)  
Folding Geometric Shapes (4-5)  
Measurement Devices/Containers (length, volume, capacity, and equivalencies)  
Pattern Blocks  
Precision Balance with Weights  
Protractors (4-5)  
Rulers  
Square Tiles  
Square (wooden) Cubes  
Tangrams  
Thermometers  
Unifix Cubes

# **Linking Mathematics and Children's Literature**

## **Benefits of the Literature Connection**

Linking mathematics instruction to children's literature has become increasingly popular in recent years for a variety of reasons. The math - literature connection motivates students, generates interest in math, helps students connect mathematical ideas to their personal experiences, accommodates children with different learning styles, inspires mathematical investigations, promotes mathematical reasoning, helps bring meaning to abstract math concepts, places math ideas in a cultural context, and provides a context for using mathematics to solve problems.

## **Ways to use Children's Literature in Teaching Mathematics**

Many children's books are explicitly about mathematics, such as books about counting or shapes while other books have mathematics embedded within a larger context. These books are generally not perceived as "math books," but mathematics appears as a natural element within stories, problems, personal vignettes, or cultural events. Welchman-Tischler (1992) has classified the ways to use such books as follows:

1. To provide a context or model for an activity or investigation with mathematical content.
2. To inspire a creative mathematics experience for children.
3. To pose an interesting problem.
4. To prepare for a mathematics concept or skill.
5. To develop or explain a mathematics concept or skill.
6. To review a mathematics concept or skill.

Though any given book could likely be used in multiple ways, the common element in these various approaches is the intent to use literature to provide vicarious mathematical experiences based on real problems or situations of interest to teachers and students.



# Integrating Children’s Literature and Math

<b>NUMBERS AND OPERATIONS</b>
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<b>COUNTING AND NUMBER SENSE</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Annie’s One to Ten</i>	Annie Owen	The illustrations in this book show all the different combinations of objects that can be grouped together to equal 10. Children can write addition and subtraction number sentences for each page. In groups, have children list all the different ways they can think of to make 10.
<i>Anno’s Counting Book</i>	Mitsumasa Anno	Provide counters in two different colors and suggest to the children that they use one color to count the girls and the other for the boys. Have them put out counters to represent the little people in the house on the first page. How many people are there altogether? Continue in this way throughout the book.
<i>Arctic Fives Arrive</i>	Elinor J. Pinczes	In this book, groups of five animals keep arriving on an iceberg to view the northern lights. After viewing the northern lights, the animal groups leave the ice in reverse order. Great for reinforcing skip counting by 5’s.
<i>“Band-Aids” in Where the Sidewalk Ends</i>	Shel Silverstein	Ask students to count all of the Band-Aids and determine the total number. Challenge them to figure out how many Band-Aids there would be if they added the “box full of thirty-five more.”
<i>The Baseball Counting Book</i>	Barbara Barbieri and Brian Shaw	On each page, starting with zero and ending with twenty, children see how each number corresponds to baseball.
<i>Bat Jamboree</i>	Kathi Appelt	Students can count, add, and subtract the bats in the jamboree.
<i>The Cheerios Counting Book</i>	Barbara Barbieri McGrath	Students can estimate and count Cheerios.
<i>City by Numbers</i>	Stephen T. Johnson	Students can search for the numbers 0-21 hidden in the urban landscape of New York City.
<i>Counting Crocodiles</i>	Judy Sierra and Will Hillenbrand	A clever monkey uses her ability to count to outwit the hungry crocodiles that stand between her and a banana tree on another island across the sea. Students can count the crocodiles in the story.

<i>Counting on Frank</i>	Rod Clement	One of the facts shared in the book is that only ten humpback whales would fit in the narrator's house. Ask students to estimate how many humpback whales would fit in their houses.
<i>The Crayon Counting Book</i>	Pam Munoz Ryan and Jerry Pallotta	Students can count and sort colors and investigate odd and even numbers.
<i>Emily's First 100 Days of School</i>	Rosemary Wells	On the 100th day of school, the children share something related to 100. One child brings 100 pieces of candy corn; another runs 100 yards. Emily worries about not having 100 things but realizes that she has written a letter to her family about what she has learned and has included 100 kisses (X's).
<i>Even Steven and Odd Todd</i>	Kathryn Cristaldi and Henry Morehouse	Even Steven, who likes everything to come in even numbers, is upset with the arrival of Cousin Odd Todd. An activities and games section is included.
<i>Feast for 10</i>	Cathryn Falwell	Numbers from one to ten are used to tell how members of a family shop work together to prepare a meal.
<i>Frogs Jump: A Counting Book</i>	Alan Brooks	This counting book gives children practice in matching number symbols to equivalent numbers of animals as well as providing practice in adding and subtracting the number 1.
<i>From One to One Hundred</i>	Teri Sloat	People and animals introduce the numbers one through ten and then count by tens up to 100.
<i>The Grapes of Math</i>	Greg Tang	Illustrated riddles introduce strategies for solving a variety of math problems using visual clues.
<i>Great Estimations</i>	Bruce Goldstone	This book will show students how to train their eyes and their minds to make really great estimations—by making estimating into a game.
<i>Harriet Goes to the Circus</i>	Betsy Maestro and Giulio Maestro	Harriet wakes up early to go the circus because she wants to be first in line. This story is useful for teaching ordinal numbers.
<i>How Many Seeds in a Pumpkin?</i>	Margaret Mcnamara	This book can be used around Halloween or Thanksgiving. Students can predict, estimate, and then count seeds in a pumpkin.
<i>How Many Snails? A Counting Book</i>	Paul Giganti and Donald Crews	This story provides the opportunity for students to count groups of objects and also to sort things by characteristics. On the last page, the author wonders how many stars there are. This is a good opportunity to talk about large numbers and even introduce the concept of infinity.

<i>The Hundred Penny Box</i>	Sharon Bell Mathis	Michael loves his great-great-aunt Dew, even if she can't always remember his name. He especially loves to spend time with her and her beloved hundred penny box, listening to stories about each of the hundred years of her life.
<i>The Icky Bug Counting Book</i>	Jerry Pallotta and Ralph Masiello	Of particular interest in this book is that the numbers extend to 26, introducing twenty-six different insects. The extensions suggest ideas for class-made books.
<i>The King's Commissioners</i>	Aileen Friedman	Have students identify the many ways to count the King's Commissioners in the book. Encourage them to comment on the various counting methods used to get to 47. Discuss whether or not the ways presented made it easier or harder to find the total number of commissioners.
<i>Let's Go Visiting</i>	Sue Williams	Students can figure out the total number of animals that the little boy saw.
<i>The M&amp;M's Counting Book</i>	Barbara Barbieri McGrath	Give students a small bag of M&M's and have them estimate and count how many are in the bag. Show a large bag of M&M's and have them use their findings to estimate how many are in the bag. Children can also sort the candy by color and count the number of each.
<i>Marvelous Math</i>	Lee Bennett Hopkins	Marvelous Math is a look at the sometimes surprising ways math is part of our daily's life. The poems cover a wide range of topics from multiplication, division, and fractions to time, counting and measurement.
<i>Math Curse</i>	Jon Scieszka	When the teacher tells her class that they can think of almost everything as a math problem, one student acquires a math anxiety which becomes a real curse. Helps students see the prevalence of math in their everyday lives.
<i>Math for All Seasons</i>	Greg Tang	This book encourages students to find the sum without counting one by one. Challenges students to solve problems by looking for patterns, symmetries, and familiar number combinations.
<i>Moira's Birthday</i>	Robert Munsch	Moira's parents are expecting six of her friends to come to her birthday party but 200 people show up. Moira orders 200 pizzas and 200 birthday cakes, but the pizza parlor and bakery can only order 10 right away. Ask students to talk about whether she needed to order that much food. How many pizzas would be needed for each guest to have 2 pieces? What other information is needed? Students could also create birthday graphs after reading this book.

<i>More or Less</i>	Stuart J. Murphy	Eddie has a booth at the school fair, guessing people's ages. He hasn't guessed wrong yet, but if he does, he gets dunked. This book gives students the opportunity to compare whole numbers and understanding what's more and what's less.
<i>More than One</i>	Miriam Schlein	In this book, readers learn that one can be more than one, as when "one pair of shoes is two shoes." Throughout the book, this idea is expanded upon to show that the number one can represent other numbers.
<i>Mouse Count</i>	Ellen Stoll Walsh	Ten mice outsmart a hungry snake. Helpful for counting 1 through 10.
<i>My Even Day</i>	Doris Fisher	This book continues where "One Odd Day" left off. The boy wakes up to discover that life is back on an "even" keel. Mom has two heads; he sees eight beds; he's served four flapjacks; he's got two left shoes; all this before he even gets to school!
<i>The Napping House</i>	Audrey Wood	Students can figure out the total number of feet in the story. Challenge them to figure out the total number of toes, fingers, or fingers and toes.
<i>Numbers</i>	Henry Arthur Pluckrose	This book helps students see that numbers are all around and are used everyday.
<i>One Duck Stuck</i>	Phyllis Root	Challenge the students to figure out how many animals came to help the one duck stuck in the muck.
<i>One Gorilla</i>	Atsuko Morozumi	The first page says, "Here is a list of things I love." Have students figure out how many things the author loved altogether. Allow them to use a variety of strategies to solve the problem.
<i>One Hundred is a Family</i>	Pam Ryan	Groups making up many different kinds of "families" introduce the numbers from one to ten and then by tens to one hundred.
<i>One Hundred Ways to Get to 100</i>	Jerry Pallotta	Teaches counting by ones, twos, fours, tens, twenties, and so on...
<i>1 Hunter</i>	Pat Hutchins	Challenge students to figure out the total number of animals seen by the hunter.
<i>One is a Snail, Ten is a Crab</i>	April Pulley Sayre and Jeff Sayre	In this story, the authors count the number of feet different animals have. The book begins with a snail's one foot, and then moves on to 2 for humans, 4 for dogs, 6 for insects, 8 for spiders, and 10 for crabs. Each odd number is represented by the even-numbered animal plus one snail. After 10, the numbers go by 10s to 100, with the number shown in two ways, for example: "70 is seven crabs- or ten insects and a crab. 80 is eight crabs - or ten spiders."

<i>One Lonely Sea Horse</i>	Saxton Freymann and Joost Elffers	Students can figure out the total number of sea creatures that the lonely sea horse finds while looking for friends.
<i>One Odd Day</i>	Doris Fisher and Dani Sneed	One morning, a boy awakens to find that everything is literally odd. The numbers on his clock read 1, 3, 5, 7, 9, and 11; his shirt has three sleeves and his dog has five legs. At school, the calendar only has odd numbers. His mother tells him to grab seven bananas before catching bus number nine. Students can search for all the odd numbers hidden throughout the pages of this vivid book.
<i>One Riddle, One Answer</i>	Lauren Thompson	After hearing the story, students can first try to solve Aziza’s riddle themselves, then they can write their own number riddles for their peers to solve.
<i>One Watermelon Seed</i>	Celia Barker Lottridge	This is a counting book that not only gives children the opportunity to count from 1 to 10, but also from 10 to 100.
<i>Only One</i>	Marc Harshman	“There may be 500 seeds, but there is only one pumpkin.” After reading this story, give students the opportunity to discuss things that come in groups of various sizes. They could write their own class book modeled after this one.
<i>A Place for Zero</i>	Angeline Sparagna LoPresti	As Zero searches to find his place, he learns of his additive and multiplicative identities, and then he establishes place value.
<i>Popcorn</i>	Frank Asch	Animated story available online at <a href="http://frankasch.com">frankasch.com</a> . Have kids estimate how many kernels of unpopped and popped corn would fit into various containers. Pose the question: “Sam’s friends brought 12 cans of popcorn to the party. The popcorn cans held 3 ounces of unpopped corn each. How many ounces of unpopped corn did the guests bring altogether?”
<i>Roar! A Noisy Counting Book</i>	Pamela Duncan Edwards and Henry Cole	A lion cub meets one red monkey, two pink flamingos, three orange warthogs, and so on but his loud roar keeps scaring them away. Finally, he meets the right playmates: nine other roaring cubs. Students can figure out the total number of animals the lion cub meets. For an extra challenge, they can figure out the number of feet on all the animals the lion cub meets.
<i>Rock It, Sock it, Number Line</i>	Bill Martin, Jr.	Introduces the numbers one through ten as vegetables and numbers dance together at the king's and queen's garden party before jumping into the soup to be eaten by a crowned boy and girl.
<i>Six-Dinner Sid</i>	Inga Moore	Students can investigate how many dinners Sid ate in one week when he lived on Aristotle Street.

<i>Stay in Line</i>	Teddy Slater	Simple math concepts are woven into a story about twelve kids and their trip to the zoo. Students can investigate how many different ways a dozen children can be grouped. What if there were two dozen children.
<i>Ten Black Dots</i>	Donald Crews	Challenge students to figure out how many black dots are on all of the pages altogether. Students can also create their own number books similar to this one.
<i>Ten Little Rabbits</i>	Virginia Grossman	Have students figure out how many rabbits there are altogether.
<i>Two Ways to Count to Ten</i>	Ruby Dee	Challenge kids to investigate ways to count to other numbers. How many ways are there to count to 12, 24, etc.? Why are there only two ways to count to 11, 13, 15, etc.? Great springboard for discussion on prime numbers, factors, and patterns.
<i>When Sheep Cannot Sleep: The Counting Book</i>	Satoshi Kitamura	When Wooly the sheep suffers from insomnia, he goes for a walk and gets into just about everything. Each illustration features objects for children to count.
<b>LARGE NUMBERS</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Can You Count to a Googol?</i>	Robert Wells	Introduces the concepts of very large numbers up to a googol.
<i>G is for Googol</i>	David M. Schwartz	Explains the meaning of mathematical terms which begin with the different letters of the alphabet from abacus, binary, and cubit to zillion.
<i>A Grain of Rice</i>	Helena Clare Pittman	Challenge students to figure out how many grains of rice Rani received in all. Calculators will be helpful. The answer is 1,073,741,823 – more than 1 billion grains of rice. Have students discuss how the number increased so quickly in a short amount of time.
<i>How Much is a Million</i>	David M. Schwartz	Have students actually make one million. Have them each draw 100 stars and hang the sheets of paper on the wall. This will take a long time so they may wish to enlist other classes to help.
<i>If you Hopped Like a Frog</i>	David M. Schwartz	Did you know that a frog can jump 29 times its body length? Or that an ant can lift an object 50 times its weight? Introduces the concept of ratio by comparing what humans would be able to do if they had bodies like different animals.
<i>A Million Dots</i>	Andrew Clements and Mike Reed	This book helps children see what a million actually looks like.

<i>A Million Fish...More or Less</i>	Patricia C. McKissack	This story can lead to a discussion of large numbers and an activity in which students can investigate dividing numbers in half. Pose the problem: "If Hugh Thomas had five hundred thousand fish and Mosley the raccoon took half, how much would he have left?"
<i>Millions of Cats</i>	Wanda Gag	Have students come up with a plan to figure out how much space one million cats would occupy.
<i>On Beyond a Million</i>	David Schwartz	Explores counting by powers of ten all the way up to the largest numbers that exist.
<b>ADDITION/SUBTRACTION</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Annie's One to Ten</i>	Annie Owen	Have students write addition and subtraction number sentences for the items on each page.
<i>Bennie's Pennies</i>	Pat Brisson	Benny sets off in the morning with five shiny new pennies to spend and eventually buys something for his mother, brother, sister, dog, and cat. Give students 5 pennies and let them "spend" their pennies as the story is read. This book helps students understand the concept of subtraction.
<i>Centipede's 100 Shoes</i>	Tony Ross	Ask students to figure out if the author figured correctly and if the little centipede really did sell all one hundred shoes and forty-two socks.
<i>Counting Wildflowers</i>	Bruce McMillan	This book uses beautiful photographs of wildflowers to illustrate the concepts of addition and subtraction up to 20.
<i>Domino Addition</i>	Lynette Long, Ph.D.	Have students place beans out to match the number of dots on each half of the domino. Then they can total the beans and write the complete addition number sentence for each example. Students can play "Domino War" similar to the game of War played with playing cards. The game can be varied by multiplying the dots on each side of the domino played.
<i>Five Little Monkeys Jumping on the Bed</i>	Eileen Christelow	This book is great for counting and subtraction. Give students monkey cutouts and let them act out the story as it is read.
<i>How Many Feet in the Bed</i>	Diane Johnston Hamm	Questions to investigate: How many feet were in the bed? How many feet are in the class? Introduce the possibility that a pet dog or cat could also be in the bed. If there were 10 feet in the bed, for example, there could be 5 people, or 3 people and one cat, or 1 person, 1 cat and 1 dog. Have the children figure different possibilities for 12 feet in the bed.

<i>Mission Addition</i>	Loreen Leedy	Miss Prime and her animal students explore addition by finding many examples in the world around them. Challenge students to find examples of addition around the classroom, school and at home.
<i>The \$1.00 Word Riddle Book</i>	Marilyn Burns	This book serves as a springboard for providing students useful practice with addition as they first figure the values of their names and then search for one-dollar words.
<i>One Less Fish</i>	Kim Michelle Toft and Allan Sheather	Give students 12 goldfish crackers and have them eat one fish cracker each time one fish disappears. Write the subtraction problem on the board. Have students create their own fish subtraction stories.
<i>1, 2, 3 to the Zoo</i>	Eric Carle	Each car on the train has one more zoo animal than the one before, from the first car with an elephant to the last with ten birds.
<i>Quack and Count</i>	Keith Baker	In this counting and adding book, a family of ducklings shows sums that add up to seven. After examining all the ways to make 7, students can be challenged to find ways to make 8, 9, 10, etc.
<i>Subtraction Action</i>	Loreen Leedy	Miss Prime and her class learn how to use subtraction to solve problems as they get ready for the school fair. Each problem is first presented as a word problem such as, "If we have ten tickets to the fair, and our class uses six of them, how many tickets will be left over?" The problem is then written out as a subtraction equation and completed systematically.
<i>10 for Dinner</i>	Jo Ellen Bogart	It's Margo's birthday and she invites 10 friends to the party. Children can think about combinations of numbers that add up to 10. The children can add birthday gifts, party food, etc. Students can write a class book modeled after this one.
<i>Ten Friends</i>	Bruce Goldstone	Challenge students to write as many different number sentences as they can that show different combinations of numbers that add to ten. This book is available at Scholastic.com.
<i>Ten, Nine, Eight</i>	Molly Bang	In this story, a young girl and her father count their ways through nightly rituals of bath, stuffed animals, dressing, and good night kisses. Students can count objects as the book is read. This book also helps children begin to understand the concept of subtraction.
<i>Ten Sly Piranhas: A Counting Story in Reverse</i>	William Wise	Ten sly piranhas are swimming in a river, but one at a time they disappear, until there is only one left. Great springboard for introducing subtraction.



<i>Ten Terrible Dinosaurs</i>	Paul Strickland	A group of rollicking dinosaurs counts down from ten to one. Great for introducing subtraction.
<i>12 Ways to Get to 11</i>	Eve Merriam	Combinations of items add up to 11. This is a good springboard for discussion of different combinations of numbers. Have their students come up with their own "number stories." This book provides a good alternative way of thinking about addition. Students can make a book modeled after this one, but use a number other than 11.
<b>Multiplication/Division</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Amanda Bean's Amazing Dream</i>	Cindy Neuschwander	After reading the story, revisit each illustration and talk with students about different ways to count the objects. Have students write their own multiplication stories. Additional lesson ideas and investigations are provided at the end of the book.
<i>Anno's Mysterious Multiplying Jar</i>	Mitsumasa Anno	Have students use calculators to verify the number of mountains, kingdoms, villages, etc. there are altogether.
"Beasts of Burden" in <i>The Man Who Counted: A Collection of Mathematical Adventures</i>	Malba Tahan	In this story, the narrator and Beremiz, wise mathematician, can't figure out how to divide an inheritance of thirty-five camels between three brothers. Have students work together to come up with a possible solution that Beremiz might use to divide the camels.
<i>Bunches and Bunches of Bunnies</i>	Louise Mathews	The book shows 81 bunnies in one classroom at school. Have them investigate how these bunnies could be split equally into smaller classes. Other problems to pose include "There are 25 bunnies shown at the ball. Four are in the band. If the rest are dancing, how many are dancing?" "At the beach, eight bunnies find shells. If each of these bunnies finds four shells, how many seashells are found altogether by these bunnies?" "When all the bunnies gathered together at the family reunion, there are 144 bunnies. One pound of rabbit pellets will feed approximately three bunnies each day. How many pounds of rabbit pellets would be necessary to feed all 144 bunnies?"
<i>The Doorbell Rang</i>	Pat Hutchins	If the children split all of Grandma's cookies among themselves, how many cookies would they get apiece? If each child in the story had one cup of milk with the cookies, how many quarts or gallons of milk would they need?

<i>Each Orange Has Eight Slices</i>	Paul Giganti and Donald Crews	This book provides an opportunity for students to think about multiplication in a way that makes sense. Students can write their own multiplication books similar to this one.
<i>The 512 Ants on Sullivan Street</i>	Carol A. Losi	Ants at a picnic keep doubling until they steal all the food. If the pattern continues, how many ants would be needed for the next three food items? Find the total number of ants each time a new food is added to the story.
<i>The Great Divide</i>	Dayle Ann Dodds	This book demonstrates the basic principle of division. Eighty contestants start out on bikes; they come to a fork in the path and half go left. The other 40 keep going by boat, until they reach a whirlpool where half of them are again knocked out of the race. This continues until only five contestants are left, leaving a tricky problem to be solved.
<i>The King's Chessboard</i>	David Birch	Challenge students to figure out how many grains of rice the wise man would get after all 64 squares were covered. Have kids investigate what is happening to the numbers each day.
<i>Minnie's Diner: A Multiplying Menu</i>	Dayle Ann Dodds	Ask students to talk about how many specials would be on the tray if there was a Grandpa McFay that came in at the end of the story. How would the story be different if there were more brothers?
<i>One Hundred Hungry Ants</i>	Elinor J. Pinczes	Talk about the different ways the littlest ant arranged his friends. Choose a new number (12, 24, 60, etc.) of ants and have students think about all the ways they could arrange themselves into rectangles to get to the picnic. Have them investigate what would happen if there were 29, 57, 105, etc. ants.
<i>One of Each</i>	Mary Ann Hoberman	In the story, Oliver solved the problem of not having enough fruit for his new friends by cutting slices of fruit and sharing them. This story is a good springboard for division.
<i>A Remainder of One</i>	Elinor J. Pinczes	Have students figure out how many ways 72 bugs could line up in equal lines? Try this with other numbers, including odd numbers. This is a good illustration of factors and prime numbers.
<i>Sea Squares</i>	Joy N. Hulme	Use tiles to interpret the pairs of numbers geometrically. Demonstrate with the page that shows 4 seals with 4 flippers each. Show the children how to arrange 16 tiles into a square array and tell them that 16 is therefore called a square number. Have the children use square tiles to check that the larger of each of the other pairs of numbers is also a square number.

<i>17 Kings and 42 Elephants</i>	Margaret Mahy	Explore the concept of division with remainders by having students solve the problem of how 17 kings could share the work of caring for 42 elephants. Other math questions – How many footprints did they all make in the jungle? How many ears and tails did they have altogether?
<i>10 Bears in My Bed</i>	Stan Mack	If ten bears are in the bed, how many feet are in the bed? How many toes?
<i>2 x 2 = Boo!</i>	Loreen Leedy	Students can create scenes like those in the book and create multiplication problems to go with their scenes. For example, they could create three bats with two eyes each and ask how many eyes do the bats have?
<i>What Comes in 2's, 3's, &amp; 4's?</i>	Suzanne Aker	Great springboard for introducing multiplication.
<b>Fractions/Decimals/Percents</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Apple Fractions</i>	Jerry Pallotta and Rob Bolster	The authors use a variety of different apples to teach kids all about fractions. Students can figure out how to share a certain number of apples among the whole class and tell what fraction of the apple each child would get to eat.
<i>Eating Fractions</i>	Bruce McMillan	This book shows various foods as wholes and then as halves, thirds, fourths, etc.
<i>Fraction Action</i>	Loreen Leedy	In this book, the hippo teacher teaches her students about halves, thirds and fourths, and about sets, fair shares, and fractions in money.
<i>Fraction Fun</i>	David Adler	This book introduces four fraction activities.
<i>Full House: An Invitation to Fractions</i>	Dayle Ann Dodds	Readers will be inspired to do the math and discover that one delicious cake divided by five hungry guests and one hospitable hostess equals a perfect midnight snack at the Strawberry Inn. Great introduction to fractions.
<i>The Hershey's Milk Chocolate Bar Fractions Book</i>	Jerry Pallotta	This book uses chocolate bars to help students understand the concept of fractions.
<i>Inchworm and a Half</i>	Elinor J. Pinczes	What's a fraction? A puzzled inchworm enlists the aid of 1/2-inch, 1/4-inch, and 1/4-inch worms in her quest to measure all the vegetables in their garden.
<i>My Half Day</i>	Doris Fisher	The main character awakens to discover that his hair is half long and half buzzed and then discovers his milk is one-third gooey paste. This book helps students learn about a variety of fractions as the main character makes a trip to Camp Fraction.

<i>Piece = Part = Portion: Fractions = Decimals = Percents</i>	Scott Gifford	This book introduces the idea that fractions, decimals, and percents are different ways of saying the same thing.
<i>The Wishing Club: A Story about Fractions</i>	Donna Jo Napoli	Petey wishes on the first star he sees, but instead of the dollar he hopes for, he gets only a quarter. The next night his little brother Joey wishes for a cookie - and gets just half of one. Try as they might, the children each wind up with a fraction of what they wished for.

<b>ALGEBRA</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Anno's Magic Seeds</i>	Misumasa Anno	Students can search for patterns to determine how many seeds Jack gets over ten years and then use their patterns to think about how to predict the number of seeds after longer periods of time.
<i>The Little Scarecrow Boy</i>	Margaret Wise Brown	This story is full of patterns and repetitions in threes and sixes.
<i>Pattern Bugs</i>	Trudy Harris	By using the sounds in the poems, the details of the pictures, and the various blocks of color that frame the initial spread, children can find and identify repetitive patterns. Each picture contains six patterns that match that picture's border.
<i>Pattern Fish</i>	Trudy Harris	Brightly colored fish inhabit a world of patterns, beginning with the simplest AB pattern and growing increasingly complex. Upon closer inspection, the patterns can be seen throughout, as pictures both express and reinforce the pattern of the words. For example, the ABB pattern introduced by an eel is echoed not only on the creature's body (stripe-dot-dot), but also in the bubbles that issue from its mouth (large-small-small), the underwater reeds (short-tall-tall) and plants (curve-star-star), and borders of the page (yellow-red-red).
<i>Patterns are Everywhere!</i>	Dominick and Alan Adunagow	This book helps children find patterns in their everyday lives. After reading the story, have students look for patterns in the classroom or in the school as they walk to lunch, art, etc.
<i>Patterns Everywhere</i>	Julie Dalton	This book helps children find patterns all around the world and in their own backyards. After reading the story, have students look for patterns in the classroom or in the school.

<i>Patterns in Peru</i>	Cindy Neuschwander	The characters in this story must use their understanding of patterns and sequences to locate the lost city—and the way back.
<i>Rabbits Rabbits Everywhere: A Fibonacci Tale</i>	Ann McCallum	Students can investigate the patterns in the story as the rabbits multiply.
<i>Rooster's off to see the World</i>	Eric Carle	Show students the pattern of pictures of the animals that appear on the inside front and back covers. Ask the students to figure out how many animals altogether set out to see the world.
<i>Sort it Out</i>	Barbara Mariconda	Children who are studying patterns and sorting will find this book interesting, especially because the packrat has so many ways to sort objects in addition to using color, size, and shape.
<i>The Talking Cloth</i>	Rhonda Mitchell	Many patterned designs decorate the “Talking Cloth” as well as the pages of the book.
<i>Two of Everything</i>	Lily Toy Hong	Challenge the students to solve the problem of how much money the couple would have at the end of ten days if they threw in one nickel each day. How many nickels would they have at the end of 10 days, 25 days, 100 days? Encourage them to create a T-chart and look for patterns. This is also a great book to use to introduce students to the concept of doubles.

## MEASUREMENT

### LENGTH AND WEIGHT

Title	Author	Annotations/Lesson Ideas
<i>Counting on Frank</i>	Rod Clement	One of the facts shared in the book is that a gum tree grows about six and one-half feet every year. Have students figure out how tall they would be if they grew at the same rate and make a representation of how tall they would be.
<i>Flat Stanley</i>	Jeff Brown	A large bulletin board fell on Stanley and flattened him. When he went to the doctor, the nurse took his measurements and discovered he was 4 feet tall, 1 foot wide and ½ inch thick. Construct Stanley according to his new proportions.
<i>Hershey's Milk Chocolate: Weights and Measures</i>	Jerry Pallotta	Introduces various measures of weight, size, and volume using Hershey's brand candies and other products. Students can weigh and measure Hershey chocolate bars.

<i>How Big is A Foot?</i>	Rolf Myller	Have students write a letter to the apprentice offering him advice. This book is helpful for helping students realize the need for standard units of measurement.
<i>How Tall, How Short, How Faraway</i>	David A. Adler	Introduces several measuring systems such as the Egyptian system, the inch-pound system, and the metric system.
<i>Inch by Inch</i>	Leo Lionni	Use as a springboard for investigating measurement. Students can estimate and then measure things in the room using one-inch tiles or rulers. Encourage students to discuss why it is more difficult to measure taller, wider, or longer items using the one-inch tiles or rulers. Have them consider better tools to use to measure the longer items.
<i>Is a Blue Whale the Biggest Thing There Is?</i>	Robert E. Wells	Pose the question: "How many fourth graders do you think it would take to equal the blue whale's length of 100 feet?"
<i>Inchworm and a Half</i>	Elinor J. Pinczes	Several small worms use their varying lengths to measure the vegetables in a garden.
<i>Jim and the Beanstalk</i>	Raymond Briggs	Show the page that shows the giant sitting at his table, reading a little book. "On this page it looks as if the giant's thumb is about the size of the book. If his thumb is the size of a book, how long is his whole hand?" Have student's estimate the length of the giant's hand. Then pose the problem: "If the giant's hand is about thirty inches long, how tall is the giant?"
<i>Length</i>	Henry Arthur Pluckrose	Photographs and text introduce the concept of length and how to measure it.
<i>Measuring Penny</i>	Loreen Leedy	Lisa learns about the mathematics of measuring by measuring her dog Penny with all sorts of units, including pounds, inches, dog biscuits, and cotton swabs.
<i>Millions to Measure</i>	David Schwartz and Steven Kellogg	Marvelosissimo the Magician explains the development of standard units of measure, and shows the simplicity of calculating length, height, weight, and volume using the metric system.
<i>"One Inch Tall" in Where the Sidewalk Ends</i>	Shel Silverstein	Great poem to use to help students understand the concept of one inch. This poem could be a springboard for having students generate common referents for an inch.
<i>Spaghetti and Meatballs for All</i>	Cindy Neuschwander	Have students re-create the book by using square tiles to represent the tables. Have them determine the perimeter and area of the different table arrangements.

<i>Twelve Snails to One Lizard, a Tale of Mischief and Measurement</i>	Susan Hightower	Milo is desperate to measure a log that he will use to patch a hole in his dam. Milo and Bubba try to measure the log, using one animal after another as a measuring tool.
<i>Weight</i>	Henry Arthur Pluckrose	Photographs and text introduce the concept of weight and how to measure it.
<i>Who Sank the Boat?</i>	Pamela Allen	The reader is invited to guess who causes the boat to sink when five animal friends of varying sizes decide to go for a row. Great introduction to the concept of weight.
<i>Zachary Zormer: Shape Transformer</i>	Joanne Anderson	This book is a good introduction to the concepts of area, perimeter, length, and width. Each Friday, Ms. Merkle has her students share objects as part of a mathematical show-and-tell. Zachary has a tendency to forget and must use his ingenuity to meet the assignment's requirements. One week, he uses a piece of paper to create a Moebius Strip, which serves as something to measure. The following Friday, he uses a scrap of paper to make an expanding frame for his lesson on perimeters. At the end of the book, each of Zachary's three transformations is explained in well-detailed steps so children can try them on their own.

### VOLUME/CAPACITY

<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Cook-a-Doodle-Doo!</i>	Janet Stevens and Susan Stevens Crummel	This book will help students discover that learning to measure is an important part of cooking and baking.
<i>A House for Birdie</i>	Stuart J. Murphy	This story about helping out a friend explains the math concept of capacity -- what will fit in a container of a particular shape and size.
<i>Me and the Measure of Things</i>	Joan Sweeney	Introduces young readers to the units of measure. What's the difference between a cup and an ounce? What gets measured in bushels and when do you use a scale? Illustrations teach children the differences between wet and dry measurements, weight, size, and length.
<i>Pigs in the Pantry: Fun with Math &amp; Cooking</i>	Amy Axelrod	Measurement concepts are explored as Mr. Pig and the piglets try to cook Mrs. Pig's favorite dish to cheer her up when she's sick. Includes a recipe for chili.

<b>TIME AND MONEY</b>		
<b>Title</b>	<b>Author</b>	<b>Annotations/Lesson Ideas</b>
<i>Alexander, Who Used to Be Rich Last Sunday</i>	Judith Viorst	Read the book in its entirety. Then read it again stopping each time Alexander spends some of his dollar and ask students to figure out how much money he has left. Let them use play or real money if possible.
<i>All in a Day</i>	Mistumasa Anno	Illustrations show the similarities and differences in children and their activities in eight different parts of the world throughout one 24-hour day.
<i>Annabelle Swift, Kindergartner</i>	Amy Schwartz	Have students use information in the book to figure out how many cartons of milk Annabelle's class purchased. Students can also figure out how much it would cost to buy milk for their own class. For a measurement activity, they could figure out how many pints, quarts, and/or gallons of milk that their whole class would drink if everyone purchased and drank all of their milk at lunch.
<i>Arthur's Funny Money</i>	Lillian Hoban	Arthur decides to charge \$.25 to wash a bicycle. Ask students to figure out how many bicycles he will have to wash before he has \$5.25
<i>Bats Around the Clock</i>	Kathi Appelt	Click Dark, the bat-version host of American Batstand, leads the 12-hour dance program. A rhyming verse teaches children how to tell time while dancing to the oldies.
<i>The Big Buck Adventure</i>	Shelley Gill	Rhyming account of a little girl's quandary as she tries to decide what she can get with her dollar in a candy shop, toy store, deli, and pet department.
<i>Bunny Money</i>	Rosemary Wells	Have students use play money to count and spend money along with the two main characters.
<i>Caps for Sale</i>	Esphyr Slobodkina	Have students figure out how much would it cost to buy all of the caps.
<i>A Chair for My Mother</i>	Vera B. Williams	A child, her waitress mother, and her grandmother save dimes to buy a comfortable armchair after all their furniture is lost in a fire.
<i>Chicken Soup with Rice</i>	Maurice Sendak	Great resource for teaching the months of the year. Students can create a class months of the year book.
<i>Clocks and More Clocks</i>	Pat Hutchins	When the hall clock reads twenty minutes past four, the attic clock reads twenty-three minutes past four, the kitchen clock reads twenty-five minutes past four, and the bedroom clock reads twenty-six minutes past four, what should Mr. Higgins do? He can't tell which of his clocks tells the right time. Have children discuss what the problem might be.



<i>The Coin Counting Book</i>	Rozanne Lanczak Williams	This book gives children the opportunity to count and add while learning the names and denominations of all of the U.S. coins. What do you get when you add five pennies together? What coin combinations add up to a quarter?
<i>Cookie's Week</i>	Cindy Ward	Cookie gets into mischief each day of the week. This book can help young children learn the days of the week.
<i>A Dollar for a Penny</i>	Julie Glass	Penny sets up a lemonade stand to earn money for her mother's birthday card.
<i>Five Minutes Peace</i>	Jill Murphy	Mrs. Large just wants five minutes peace. She heads for the kitchen and finds exactly three minutes and 45 seconds of peace.
<i>The Go Around Dollar</i>	Barbara Johnston Adams	A story describing how a single dollar changes hands, accompanied by facts about the one-dollar bill.
<i>The Grouchy Ladybug</i>	Eric Carle	The grouchy ladybug begins the day at five o'clock in the morning and ends it around six o'clock at night. How many hours was he flying around?
<i>How Do You Know What Time it Is?</i>	Robert Wells	This book introduces the concept of time and how we measure it.
<i>If You Made a Million</i>	David Schwartz	The book shows four different combinations of coins that are equivalent to a one dollar bill. Have the students list as many combinations as they can of coins and bills that are equivalent to a five dollar bill.
<i>Jelly Beans for Sale</i>	Bruce McMillan	This book introduces pennies, nickels, dimes, and quarters and the value of each; that is, each cent is worth one jelly bean. The author represents several different amounts of money with many combinations of coins. Available at Scholastic.com
<i>The Money Tree</i>	Sarah Stuart	Miss McGillicuddy notices a strange tree in her yard. Month by month, as the seasons change, it grows, faster than any normal plant, into a money tree.
<i>Monster Math School Time</i>	Grace MacCarone	From the time they get up at seven in the morning until they go to bed at eight o'clock at night, monsters spend a busy day, especially at school. Activities for students are included.
<i>Monster Money Book</i>	Loreen Leedy	The members of the Monster Club discuss money and how to manage it.
<i>My Grandmother's Clock</i>	Geraldine McCaughrean	A child, wondering why Grandma doesn't have the grandfather clock in her house repaired, learns that there are many ways to measure time.

<i>My Rows and Piles of Coins</i>	Tololwa M. Mollé	A Tanzanian boy saves his coins to buy a bicycle so that he can help his parents carry goods to market, but then he discovers that in spite of all he has saved, he still does not have enough money.
<i>On the Day You were Born</i>	Debra Frasier	Students can investigate the number of days that have passed since they were born.
<i>Once Upon a Dime</i>	Nancy Kelly Allen	Farmer Worth discovers that a special tree on his farm produces different denominations of money, depending on what animal fertilizer he uses.
<i>Only Six More Days</i>	Marisabina Russo	Students can use calendars and counters or Snap Cubes to figure out how many more days until their birthdays.
<i>P. Bear's New Year's Party</i>	Paul Owen Lewis	The first guest arrives at one o'clock and at two o'clock, two more guests arrive. Every hour after that, the number of guests that arrives matches the hour on the clock. Challenge kids to answer the question on that last page of the book, "How many guests came to the party?" This gives students the opportunity to practice counting and addition while reviewing time.
<i>Pigs Go to Market: Fun with Math and Shopping</i>	Amy Axelrod	The Pigs can't wait to throw their annual Halloween party. But when Grandpa and Grandma Pig eat all of the candy, the Pigs have to make a last minute trip to the market, where Mrs. Pig wins a free shopping spree. This book provides practice with the math skills of price and quantity.
<i>Pigs on a Blanket: Fun with Math and Time</i>	Amy Axelrod	Great story for teaching elapsed time. Students can figure out how long it actually took the pigs to drive to the beach and how much longer the drive than was than it should have been.
<i>Pigs Will Be Pigs: Fun with Math and Money</i>	Amy Axelrod	Provides an excellent way to talk about how decimals are used to represent money numerically. Also provides a context that focuses on the values of coins and bills and provides practice with adding money.
<i>Quarter from the Tooth Fairy</i>	Caren Holtzman	A boy has trouble deciding how to spend the quarter he gets from the Tooth Fairy. He learns about money and the different coin combinations that make up a quarter.
<i>"Smart" in Where the Sidewalk Ends</i>	Shel Silverstein	Have the students write a letter to the little boy offering him advice. Did the boy get a good deal like he thought he did?
<i>The Story of Money</i>	Betsy Maestro	The author gives a history of money, beginning with the barter system in ancient times, to the first use of coins and paper money, to the development of modern monetary systems.

<i>Telling Time</i>	Jules Older	This book explains the concept of time, from seconds to hours on both analog and digital clocks and from years to millennia on the calendar. A poem at the end reminds children how many seconds are in a minute, how many minutes are in an hour, and so on.
<i>365 Penguins</i>	Jean-Luc Fromental	On the first day of the new year, the mailman brings a surprise - a penguin! One by one, day by day, penguins fill the house. As they arrive, readers must recall the number of days in each month. By the end of February, they are calculating the number of penguins in all. Then Father decides to organize them, first into four groups of 15, later in boxes by the dozen, and, finally, into a cubic formation.
<i>Time</i>	Henry Arthur Pluckrose	Time is explained in a historical, future and present sense. Readers are given examples of each type of time being described. Time is also explained in the realm of seasons, day and night, and calendars, along with telling time on a clock. Both an analog clock and a digital clock are explained as well as how to read them.
<i>Time to...</i>	Bruce McMillan	This book introduces children to the concept of telling time. Photos follow a little boy's daily activities as each double-page spread advances the clock by one hour. This not only familiarizes children with different kinds of clocks, but also introduces the concept of A.M. and P.M.
<i>Today is Monday</i>	Eric Carle	Each day of the week brings a new food, until on Sunday all the world's children can come and eat it up.
<i>The Toothpaste Millionaire</i>	Jean Merrill	Have students find out the cost of toothpaste in a local store and calculate the cost per ounce.
<i>26 Letters and 99 Cents</i>	Tana Hoban	Color photographs of letters, numbers, coins, and common objects introduce the alphabet, coinage, and the counting system.
<i>The Very Hungry Caterpillar</i>	Eric Carle	Challenge students to figure out how many things the caterpillar ate in one week. They could also create a book entitled "The Very _____ following the model using days of the week for each page.
<i>What Time is It?</i>	Sheila Keenan	A child is so excited about her Grandpa's visit that she counts the hours until he arrives. Challenge students to figure out how many hours until some event is to occur in the classroom (recess, lunch, end of school day, pizza party, etc.).

<i>What Time is It, Mr. Crocodile?</i>	Judy Sierra	There is a clock face on every double page spread so children can enjoy the countdown as well as reinforcing telling time for those just learning the skill.
<i>America's Champion Swimmer: Gertrude Ederle</i>	David Adler	Students can choose a sport to investigate with an emphasis on measurement. Have them use newspapers, the Internet, magazines, and other available sources to find out what aspects of measurement are used in that sport. For example, in football both time and length are important and the game is timed in minutes and seconds.

## GEOMETRY

<b>TITLE</b>	<b>AUTHOR</b>	<b>COMMENT</b>
<i>Bear in a Square</i>	Stella Blackstone	This book is a good introduction to two-dimensional shapes. Bear looks for shapes around him.
<i>A Cloak for the Dreamer</i>	Aileen Friedman	Have students write letters to the character Misha explaining the problem with the cloak and offering solutions to the problem. Students can also make cloak patterns of their own.
<i>Color Farm</i>	Lois Ehlert	The rooster, dog, sheep, cow, pig, and other animals on a farm are made up of colorful shapes such as square, circle, rectangle, and triangle.
<i>Cubes, Cones, Cylinders and Spheres</i>	Tana Hoban	Photographs of all kinds of familiar objects depict a variety of shapes, including cubes, cones, and spheres. Follow up by having students find examples of these three-dimensional shapes in the classroom or at home.
<i>Ed Emberley's Picture Pie</i>	Ed Emberley	Shows how to cut a basic circle into arcs and curves and use the pieces to draw birds, animals, snowmen, fish, and many other objects and designs. This book could also be used to help students understand the concept of fractions. Teachers can have students make pictures and tell what fraction of the circles they used to create their designs.
<i>Ed Emberley's Picture Pie 2</i>	Ed Emberley	Shows how circles, squares, and triangles can be used to create all kinds of pictures. Students can use shapes to create their own pictures after enjoying this book.
<i>Grandfather Tang's Story</i>	Ann Tompert	Allow students to make their own tangrams and make pictures with their tangrams.
<i>The Greedy Triangle</i>	Marilyn Burns	Good book to use as a springboard for exploring shapes.
<i>Look at Annette</i>	Marion Walter	Have students use mirrors and die cuts of letters to see which of the letters of the alphabet have symmetry. Which letters have one line of symmetry and which

		ones have more than one line of symmetry? Can they find words that have symmetry (WOW, MOM, etc.)
<i>Mummy Math: An Adventure in Geometry</i>	Cindy Neuschwander	Stuck inside a pyramid with only each other, their dog Riley, and geometric hieroglyphics to help them find their way, the twins must use their math knowledge to solve the riddles on the walls and locate the burial chamber.
<i>Reflections</i>	Ann Jonas	Chronicles a child's busy day by the sea, in a forest, and then home with the hope of going to a carnival or concert. The illustrations change when the book is turned upside down. This book can be used to introduce reflections.
<i>Sam Johnson and the Blue Ribbon Quilt</i>	Lisa Campbell Ernst	After reading the book and looking at the various quilt patterns, students explore ideas in geometry as they work together to make paper quilts.
<i>The Secret Birthday Message</i>	Eric Carle	Students can find all the shapes in the pictures throughout the book.
<i>Shape</i>	Henry Arthur Pluckrose	Photographs of familiar objects introduce basic shapes of squares, circles, rectangles and triangles.
<i>The Shape of Things</i>	Dandi Daley Mackall	Great springboard to introduce shapes-circles, squares, rectangles, triangles to young children.
<i>Geometry The Village of Round and Square Houses</i>	Ann Grifalconi	Students can draw the shapes that make up the structure of their own homes and write the names of the shapes. Students can create three-dimensional structures and make their own villages.
<i>Shape Up! Fun with Triangles and Other Polygons</i>	David A. Adler	The author uses cheese slices, pretzel sticks, a slice of bread, graph paper, a pencil, and more to introduce various polygons.
<i>Shapes, Shapes, Shapes</i>	Tana Hoban	Talk about each photograph with the students, asking them to identify as many shapes as they can. After examining the photographs in the book, have the children look for and describe the shapes they see in the classroom, around the school, at home, or on walks in the neighborhood.
<i>So Many Circles, So Many Squares</i>	Tana Hoban	The geometric concepts of circles and squares are shown in photographs of wheels, signs, pots, and other familiar objects.
<i>The Tangram Magician</i>	Lisa Campbell Ernst and Lee Ernst	Have students experiment with making different shapes with the tangram pieces. Invite children to create some of the tangram figures that are illustrated in the book as well as some of their own.

<i>Three Pigs, One Wolf, and Seven Magic Shapes</i>	Grace Maccarone	In this story, the pigs meet magic animals that give them seven magic shapes to help them find their fortune. These shapes form a tangram. Notes at the end of the book include a cut-out tangram and suggestions for creating other shapes with this new tool.
<i>The Warlord's Puzzle</i>	Virginia Pilegard	After hearing the story, students use their own set of tangrams and explore making different polygons.
<i>When a Line Bends...A Shape Begins</i>	Rhonda Gowler Greene	Ask students to explain the author's meaning when she wrote, "When a line bends...A shape begins." Have students find triangles, circles, squares, octagons, diamonds, rectangles, etc. throughout the book.

## DATA ANALYSIS AND PROBABILITY

<b>TITLE</b>	<b>AUTHOR</b>	<b>ANNOTATIONS/LESSON IDEAS</b>
<i>Caps for Sale</i>	Esphyr Slobodkina	Have students investigate how many different arrangements of caps he could wear if he always wore the checkered one on his head and kept the same color hats together?
<i>Chrysanthemum</i>	Kevin Henkes	Students can create graphs using data of how many letters are in their names.
<i>Graphs</i>	Bonnie Bader	A boring family reunion provides data for charting graphs for a homework assignment.
<i>Jesse Bear, What will You Wear</i>	Nancy White Carlstrom	Use Teddy Bear die cuts and cutouts of shirts and pants to figure out how many different combinations of outfits Jesse could wear. Challenge them to add hats to the combinations. Children will be surprised by how many more combinations are available when three articles of clothing are used. See <a href="http://illuminations.nctm.org/ActivityDetail.aspx?ID=3">http://illuminations.nctm.org/ActivityDetail.aspx?ID=3</a> for an interactive activity (Bobbie Bear) that can be done using the Promethean board.
<i>No Fair!</i>	Caren Holtzman	Two children play several games of chance trying to figure out what is mathematically fair. Challenge students to create games that are mathematically fair.
<i>It's Probably Penny</i>	Loreen Leedy	Lisa's class is learning about probability. For part of her homework, she has to think of an event that will happen, one that might happen, and one that can't happen.
<i>A Three Hat Day</i>	Laura Geringe	Suppose R.R. Pottle wanted to cheer himself up on another day and put on the same three hats before

		going for his walk. But suppose that he decided to put on the hats in a different order? And then the next day he put the same three hats on in a different order again. How many days could R.R. Pottle wear those same three hats if each day he put them on in a different order?
<i>A Very Improbable Story: A Math Adventure</i>	Edward Einhorn	Waking up one morning and finding a talking cat on his head, Ethan is informed that the cat will not leave until he has won a game of probability. Challenge students to think of events that are likely, equally likely, unlikely, certain, or impossible.
<i>The Best Vacation</i>	Stuart J. Murphy	Have students select a survey question and collect data by interviewing fellow students. As they complete their surveys, have them select a way to represent the data and share it with the class.

# Technology and Mathematics

## Calculators

Calculators should be recognized as an instructional tool – similar to rulers and other manipulatives. As a result, students should have access to calculators everyday (Van de Walle, 2000). In the primary grades (kindergarten – second), the calculator is for exploration. As the students show mastery of math facts and basic skills, the calculator should be used to reinforce the facts. In the upper elementary grades (third – fifth), the calculator should be used as another strategy to learn concepts. Students should have daily opportunities to use the calculator as another learning tool, especially in other content areas that rely on math (such as science) and for topics that require the application of basic skills (calculating the mean, perimeter, etc.). After students have shown understanding and mastery of an algorithm, the calculator should be used to check work.

## Promethean and Math Instruction

As all of Rock Hill Schools' classrooms become Activclassrooms, teachers must understand the power to improve math instruction through the proper use of Promethean Boards. Here are some basic flipchart fundamentals as found on Promethean Planet.

### 1. Use proven planning strategies.

When constructing a flipchart lesson for use on the Activboard, begin by using the same lesson planning strategies you would typically use when building a lesson that is not being delivered on an interactive whiteboard. Establish lesson/activity objectives, expected outcomes, attainment targets being addressed and description of specific material to be covered during the lesson. A good lesson is a good lesson, regardless of the vehicle used for delivery, and it always begins with the aforementioned core elements.

### 2. Maximize student participation.

When considering the role of the student in the lesson, attempt to create multiple opportunities for interaction, response and feedback. You want the students actively participating in the lesson, not just serving as a passive audience to a presentation. Try to “mix up” the types of interaction as well. Students can interact verbally, come up to the board individually, work from their seats with the Activslate, or participate as a group using Activotes. A great way to keep students on task is to keep the Activotes out all the time and ask students to agree or disagree with what an individual has contributed to the discussion or lesson.



### **3. With experience comes confidence.**

It's often daunting when you see other people's advanced flipcharts and you wonder 'how will I ever achieve that?' The answer is, in time you will! Any beginner can advance their own Activboard skills by experimenting with the software and starting with the more basic tools, such as rub and reveal.

Here's an easy three-point guide to this simple, but effective, technique:

- Type some text on the page.
- Write over the text with a pen.
- Then, in the lesson, you can use the eraser tool to erase the pen, 'revealing' the text beneath...

### **4. Take advantage of what you already have.**

The new and improved resource library features more than 15,000 teaching resources including images, backgrounds, lessons, sounds, shapes, lines, grids, annotations, and flash activities...all searchable by keyword.

It also contains lesson-building templates for creating whole-group assessment pages including voting buttons, question page layouts and backgrounds. Before spending hours looking for resources elsewhere, have a look in the resource library first!

There are also thousands of pre-made flipcharts, weblinks and resource packs ready for download in the [Resource Section](#) on Promethean Planet. Planet offers teachers a place to upload their flipchart lessons and share them with others around the country. Search for ones you want and download them to use with your students. Once you download the lesson, it's yours to modify, add to, and use as you wish!

### **5. Sharpen your skills.**

Once you are familiar with Activstudio or Activprimary, you'll be ready to move on to more advanced skills and techniques. Planet's [Activtips section](#) offers quick tips for integrating tools and techniques into various curriculum areas. Inspired by user suggestions and questions, the Activtips section has something for everyone.

The [Forum](#) and [Blog](#) is another fantastic area for sharpening your skills. With users just like you looking to sharpen their skills, there's always someone ready and willing to share an idea and offer a helping hand.

### **6. Share ideas and lessons with others.**

Perhaps the greatest resource for creating top notch flipcharts is your colleagues. When logged in to the district network all district teachers, via our network servers, have access to a district wide 'drop box' where teachers can place flipchart lessons for mass

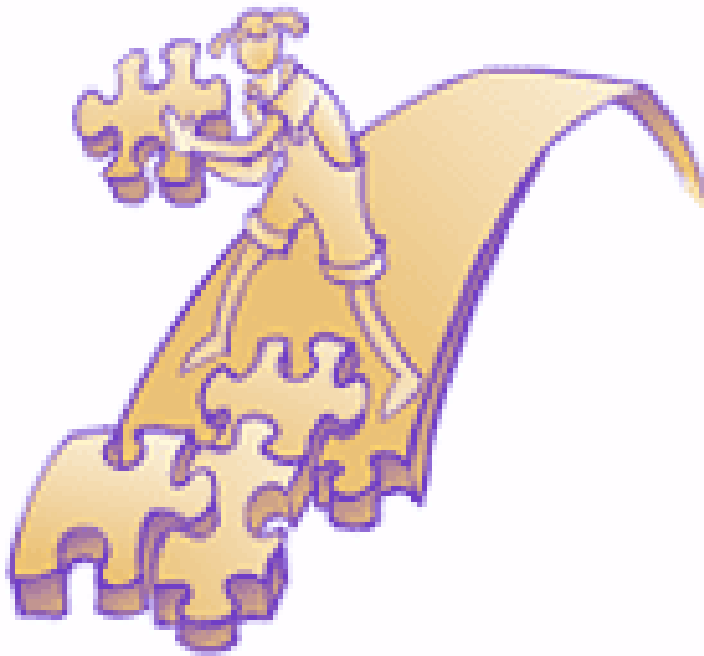
consumption. You will find an icon on your desktop labeled Promethean Flipcharts. Divide and conquer is their motto and teachers are able to share the load when it comes to lesson planning. In doing so, they also tend to learn new techniques, master new Activsoftware tools and develop consistent, proven methods for lesson construction and delivery.

The Promethean Planet online resource library is a great place to post lessons as well, particularly because they are aligned to curriculum topics and student age ranges for you.

**And don't forget...**The Planet forum is a superb place to start sharing ideas and lesson plans with other Activboard users from across the globe!

Source: [www.prometheanplanet.com](http://www.prometheanplanet.com)

# Teacher Resources



Classroom organization addresses all components of instruction – teaching strategies, student grouping assignments, and assessing (Slavin, 1989).

## **Sample Math Workshop Lesson Plan**

<b>Lesson Title</b>
<b>SUGGESTED MATERIALS:</b>
<b>PROCESS INDICATORS ADDRESSED:</b>
<b>CONTENT INDICATORS ADDRESSED:</b>
<b>MINI LESSON</b>
<b>SMALL GROUP WORK</b>  <b>Plans for Differentiation:</b>
<b>WHOLE GROUP DISCUSSION</b>

## Let's Go Visiting

**SUGGESTED MATERIALS:** *Let's Go Visiting* by Sue Williams, Linking Cubes, Promethean Board, Hundreds chart

**PROCESS INDICATORS ADDRESSED:**

Problem solving, Reasoning and Proof, Connections, Communication, Representation

**CONTENT INDICATORS ADDRESSED:**

**K-2.1** Recall numbers, counting **forward** through 99 and backward from 10.

**K-2.2** Translate between numeral and quantity through 31.

**MINILESSON**

- Set the stage for the problem-solving situation that will follow the story by asking them to think about how many animals the child in the book sees each day.
- Read the story and allow the students to comment about the book.
- Read the book again and allow students to write the numerals on the board for the quantity of animals on each page.
- On the 2 red calves' page, ask if they know how many animals they saw in the first 2 days. Show the cubes. Write this on board. Do same for three kittens' page. If kids are ready, write it in an addition problem.
- After reading the story, present the problem to be solved: How many animals did the child visit in all of the days?
- Allow kids to estimate and write estimates on board.
- Show the students the page where the child is sleeping with all the animals and ask them if they think this is all the animals. Let them try to count the animals. Ask if there is a way that they can be sure that this is all the animals that the child visited in all six days.
- Tell the kids that they are now going to go into small groups to figure out how many animals the child visited in all six days.

**SMALL GROUP WORK**

- Students will work in small groups to solve the problem. The students will use the cubes and to represent the number of animals the child sees each day. Circulate and assist kids by asking questions to probe their thinking. Be careful not to direct them toward a particular strategy. The goal is that the students work together to come up with a strategy that they understand and that makes sense to them. If a group solves the problem quickly and with ease, encourage them to use another strategy to determine the total number of cubes.

**Plans for Differentiation:** Allowing students to choose their own strategies for counting the cubes will allow them to solve the problem at the level of difficulty that they are comfortable with. Some kids will count by 1's, others who are more advanced might count by 5's or 10's and others may even use counting on or an adding strategy. Provide extra assistance for students who are having trouble getting started and/or counting the cubes.

**WHOLE GROUP DISCUSSION**

Reconvene the class and allow each group to share the strategy (s) they used to count the cubes. Show their representations on the Promethean board. Write the numerals as they count. Let the kids discuss which strategies worked best for them. Ask the kids to summarize what they felt they learned in today's lesson.

## *Quack and Count*

**SUGGESTED MATERIALS:**

Linking cubes, square tiles, other various counters, *Quack and Count* by Keith Baker

**PROCESS INDICATORS INTEGRATED:**

Problem solving, Connections, Communication, Representation, Reasoning and Proof

**CONTENT INDICATORS ADDRESSED:**

**1-3.1** Analyze numeric patterns in addition to develop strategies for acquiring basic facts.

**MINI LESSON**

Read the story. Ask students to talk about the math they see in the book. Ask them to talk about how many different ways the author made 7. List the ways as they share. Ask students to look at the list and make observations. Ask them if there is a way to use a mathematical symbol to show what the author is doing (i.e., one plus 6 is  $1 + 6$ ). Pose the question – What if there were 8 ducks? I wonder how many different ways the author could have made 8. Let students predict. Do 8 together as a whole group. Challenge students to work together to come up with all the ways to make 9, 10, and 11.

**Example of ways to make 8:**

$$7 + 1$$

$$6 + 2$$

$$5 + 3$$

$$4 + 4$$

$$3 + 5$$

$$2 + 6$$

$$1 + 7$$

Note: The book does not use zero as a way to make the sum of 7. Some students may choose to use zero. This should be accepted. Help students see how the patterns are different if zero is used.

**SMALL GROUP WORK**

Students work together to solve the problem of how many different ways to make 8, 9, 10, and 11. The teacher should encourage them to generate strategies to solve the problem that makes the most sense to them.

**Plans for Differentiation:**

Students might work with numbers less than 7. Some students may need the entire small group time to find the ways to do just one number while some students may go beyond the numbers asked of them. Allowing students to choose the tools and strategies that work best for them will allow all students to solve the problem in the way that makes the most sense for them.

**WHOLE GROUP DISCUSSION**

Students share their strategies and solutions with the whole group. Ask students to “prove” that they got all of the possibilities. How did they determine that they had all of the possibilities? List all possibilities on the board. Ask them to look for patterns. Finally, draw a T-chart on the board. As they notice patterns, ask them to predict what the next number would be. Students will begin to notice that the number of ways to make any given sum is one less than the sum. Ask them if they can predict the number of ways to make the sum of 25, 50, 100, etc. using this pattern.

**Sum      Number of Ways to Make the Sum**

7	6
8	7
9	8
10	9
11	10
12	11

## Measurement Scavenger Hunt

### SUGGESTED MATERIALS

#### For each group of 4:

rulers, yardsticks, square tiles, centimeter cubes, picture of football field, Karate belt or other item of similar length, children's book on measurement (see suggestions below)

### PROCESS INDICATORS ADDRESSED:

Problem solving, communication, representation, reasoning and proof, connections

### CONTENT INDICATORS ADDRESSED:

**2-5.3** Use appropriate tools to measure objects to the nearest whole unit: measuring length in centimeters, feet, and yards.

### MINI LESSON

- Read a children's book about measurement such as *How Big is a Foot* by Rolf Myller, *Inch by Inch* by Leo Lionni, or *Measuring Penny* by Loreen Leedy.
- Show the class pairs of straight objects such as a pencil and a football field (show a picture of this), the teacher and a crayon, or the height of a book and a Karate belt. For each pair, ask: Which is longer? Which is shorter? How can we tell? What are some things that you could use to measure these items? Allow students to search through their toolboxes and share items that they might use to measure each item. Avoid telling them directly which tool is the most efficient to use. The goal is to have them discuss, investigate, and discover as they engage in a small group scavenger hunt.
- Before beginning the scavenger hunt, briefly discuss each tool. Ask students if they can name each tool and tell the length of each tool. For example, a ruler is a foot long, a yardstick is a yard long, a square tile is an inch long, and the centimeter cubes are a centimeter long.
- Tell the class that they are going to work in their groups to go on a measurement scavenger hunt. Tell them that they may use any of the tools in their toolboxes to measure the items that they choose. Encourage them to talk together and agree upon a tool to use before measuring each item. Encourage them to use a variety of measurement tools.

### SMALL GROUP WORK

Students should work in groups of three to four to locate and measure objects decided on in advance by the teacher. They should be encouraged to use their own strategies to decide what and how to measure. Answer questions as necessary but be careful not to "tell" the students how to measure each item or what tool to use. The goal is to tell them just enough so that they can figure out the most efficient measurement tool to use for each item. Through the process of engaging in the investigation, students will begin to discover that some tools and units (inches, centimeters, or feet) are more efficient than others when measuring certain things.

**Plans for Differentiation:** Some students might choose to use square tiles if they haven't had a lot of experience with using rulers or yardsticks. It's important not to discourage this. Students should be allowed to use the tool that is easiest for them.

### WHOLE GROUP DISCUSSION

After the students have been given sufficient time to conduct the investigation, reconvene the group to share the objects chosen, tools used to measure, and measurements discovered. Ask students to talk about which tools were easier for them to use to measure each object. Through the follow-up discussion, students should begin to see that certain tools are better to use to measure certain objects. For example, we wouldn't want to use centimeter cubes or even a ruler to measure something as tall as a basketball goal. Be sure to relate these newly constructed ideas back to the ideas shared during the mini lesson. It is important to remember that the goal of this lesson is not **exact** measurement. Instead, students are being given the opportunity to build understanding of appropriate tools for linear measurement while at the same time, beginning to practice using those tools to measure.

## How Old are You?

**SUGGESTED MATERIALS:** assorted counters, Unifix/Linking Cubes, square tiles, Base 10 Blocks, Cuisenaire Rods

**PROCESS INDICATORS ADDRESSED:**

Problem solving, Connections, Communication, Representation, Reasoning and Proof

**CONTENT INDICATORS ADDRESSED:**

**3-2.10** Generate strategies to multiply whole numbers by using one single-digit factor and one multi-digit factor.

### MINI LESSON

Present the problem to be solved:

Today is Juan's birthday and he is 8 years old. How many months old is he? How many months old will he be when he turns 9?

Discuss any background knowledge needed to solve the problem. For example, students need to remember how many days in a year before they can generate strategies to solve the problem.

Explain that students are to divide into small groups and use strategies of their choosing to solve the problem.

### SMALL GROUP WORK

Students should continue to work in groups of 3-4. Students are encouraged to use their own strategies to solve the problem. As the students explore using manipulatives and other problem-solving strategies of their choosing, the teacher will facilitate by asking individual groups:

- What are some ways that your group has discovered?
- What strategies did your group use?
- Can you think of another strategy that would work?
- Could you show your strategy using a picture?
- 

**Plans for Differentiation:** Allowing students to choose the problem-solving strategy to use instead of dictating how they should solve the problem will also allow for differentiation. The problem could be made more difficult by having the students find out how many weeks or days old Juan is. Some students may need the entire block of small group time just to solve the first part of the problem.

### WHOLE GROUP DISCUSSION

After the students have been given sufficient time to generate a variety of strategies, reconvene the group to share strategies and solutions. Allow each group the opportunity to share their findings as well as new learning and connections made. Ask students if they can come up with a pictorial representation for their concrete manipulative representations. Allow them to draw their representations on the Promethean Board.



## Toothpick Triangles

**SUGGESTED MATERIALS:** toothpicks, paper, pencils

**MATHEMATICAL PROCESSES:** The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

**CONTENT INDICATORS ADDRESSED:**

**4-3.1** Analyze **numeric**, nonnumeric, and repeating patterns involving all **operations** and decimal patterns through hundredths.

**4-3.2** Generalize a rule for **numeric**, nonnumeric, and repeating patterns involving all operations.

**4-3.3** Use a rule to complete a sequence or a table.

**MINI LESSON**

- How many toothpicks does it take to make one triangle?
- Is there a way to make 2 triangles with 5 toothpicks?
- Is there a way to use 7 toothpicks to make 3 triangles?

*Present the problem to be solved:*

If it takes 3 toothpicks to make 1 triangle, 5 toothpicks to make 2 triangles and 7 toothpicks to make 3 triangles, how many toothpicks does it take to make a row of 10 triangles? 25? 50? 100?

**SMALL GROUP WORK**

Students work in groups of 2 – 4 to solve the problem. Students are encouraged to use their own strategies to solve the problem. Some may choose to draw pictures and some may choose to use the toothpicks.

**Plans for Differentiation:** Some children might actually need to lay out the entire row of triangles to solve the problem. Some may see the pattern immediately. Groups that finish quickly should be encouraged to find a rule for finding out how many toothpicks it would take to make any number of triangles. Challenge them to solve the same problem with another shape such as a square or a triangle.

**WHOLE GROUP DISCUSSION**

After the students have been given sufficient time to conduct the investigation, reconvene the group to share strategies and solutions. Allow each group the opportunity to share their findings as well as new learning and connections made. After students share strategies and solutions, show them how to organize their findings in a t-chart.

<u>Triangles</u>	<u>Toothpicks</u>
1	3
2	5
3	7
4	9

Then, ask the students to look for patterns in the chart. There are patterns in the toothpick column (increasing by 2 in each row), but the patterns between the triangle and toothpick column is what they need to see to figure out the solution for a greater number of triangles without having to actually make all of the triangles.

Encourage them to think of a rule for the chart. A rule for the chart would be to multiply the number of triangles by 2 and add 1. For example, to make 4 triangles, it would take 9 toothpicks because  $4 \times 2 + 1 = 9$ . Challenge students to figure out why this works. Another way to write this using symbols would be:

$$\triangle \times 2 + 1 = T$$

## Fractions – Unlike Denominators

**SUGGESTED MATERIALS:** fraction tiles, fraction bars, fraction pieces, color tiles, pattern blocks, grid paper, and colored pencils.

**PROCESS Standard:**

The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

**CONTENT INDICATORS ADDRESSED:**

5-2.8 Generate strategies to add and subtract fractions with like and unlike denominators.

### MINI LESSON

Ask the students to talk about how to find the LCM of 6 and 8? **LCM = 24**

Introduce and discuss the details of the first problem together. Have the students identify the key components of the problem. Have the students recognize that this is an addition problem and the fractions have different denominators.

### SMALL GROUP WORK

Students will work together in groups of 3-4 to solve the second and third problems. They should be encouraged to use “tools” from their toolboxes to determine the solution for each problem. The teacher should monitor progress as students work in their groups. Encourage students to explain how they know their solutions are accurate.

Maria plants vegetables in  $\frac{2}{3}$  of her garden and fruit in  $\frac{1}{6}$  of her garden. How much of the garden has been planted?  $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

It took Kayla  $\frac{1}{4}$  of an hour to clean her room on Saturday. Robin cleaned her room in  $\frac{3}{8}$  of an hour. How much time did they spend cleaning their rooms?  $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

Samantha wants to make a Pirate ship banner for the Fall Festival. Two-fifths of a yard of red cloth was used to make the banner, and half of a yard of black cloth was used. How much cloth did she use?  
 $\frac{2}{5} + \frac{1}{2} = \frac{4}{10} + \frac{5}{10} = \frac{9}{10}$

**Plans for Differentiation:** This is an introductory lesson, so differentiation will be the tools the students choose to use. You could also differentiate based on the students’ ability to add like denominator fractions.

### WHOLE GROUP DISCUSSION

Students should share the different strategies they chose. As they share, teachers can ask the following questions when appropriate:

For addition, what do we have to do when the fractions have unlike denominators?

Did one problem challenge you more than another problem?

What previous math concepts can you use to add fractions with unlike denominators?

How can you convince me that your strategy worked?

<b>EXEMPLARS RUBRIC</b>			
<b>Level</b>	<b>Understanding</b>	<b>Strategies, Reasoning, Procedures</b>	<b>Communication</b>
<b>Novice</b>	<ul style="list-style-type: none"> <li>There is no solution, or the solution has no relationship to the task.</li> <li>Inappropriate concepts or procedures are used.</li> </ul>	<ul style="list-style-type: none"> <li>No evidence of a strategy or procedure, or uses a strategy that does not help solve the problem.</li> <li>No evidence of mathematical reasoning.</li> <li>There were so many errors in mathematical procedure that the problem could not be solved.</li> </ul>	<ul style="list-style-type: none"> <li>There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem.</li> <li>There is no use or inappropriate use of mathematical representations (e.g., figures, diagrams, graphs, tables, etc.)</li> <li>There is no use, or mostly inappropriate use, of mathematical terminology and notation.</li> </ul>
<b>Apprentice</b>	<ul style="list-style-type: none"> <li>The solution is not complete, indicating that parts of the problem are not understood.</li> </ul>	<ul style="list-style-type: none"> <li>Uses a strategy that is partially useful, leading some way toward a solution, but not to a full solution of the problem.</li> <li>Some evidence of mathematical reasoning.</li> <li>Could not completely carry out mathematical procedures.</li> </ul>	<ul style="list-style-type: none"> <li>There is an incomplete explanation; it may not be clearly presented.</li> <li>There is some use of appropriate mathematical representation.</li> <li>There is some use of mathematical terminology and notation appropriate to the problem.</li> </ul>
<b>Practitioner</b>	<ul style="list-style-type: none"> <li>The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution.</li> </ul>	<ul style="list-style-type: none"> <li>Uses a strategy that leads to a solution of the problem.</li> <li>Uses effective mathematical reasoning.</li> <li>Mathematical procedures used appropriately.</li> </ul>	<ul style="list-style-type: none"> <li>There is a clear explanation.</li> <li>There is appropriate use of accurate mathematical representation.</li> <li>There is effective use of mathematical terminology and notation.</li> </ul>
<b>Expert</b>	<ul style="list-style-type: none"> <li>The solution shows a deep understanding of the problem, including the ability to identify the appropriate mathematical concepts and the information necessary for its solution.</li> </ul>	<ul style="list-style-type: none"> <li>Uses a very efficient and sophisticated strategy leading directly to a solution.</li> <li>Employs refined and complex reasoning.</li> <li>Applies procedures accurately to correctly solve the problem and verify the results.</li> </ul>	<ul style="list-style-type: none"> <li>There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made.</li> <li>Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem.</li> <li>There is precise and appropriate use of mathematical terminology and notation.</li> </ul>

*How to Assess Problem-Solving Skills in Math* by Bena Kallick and Ross Brewer, Scholastic Professional Books, 1997

<b>MATH STUDENT RUBRIC</b>			
<b>Level</b>	<b>Understanding</b>	<b>Strategies, Reasoning, Procedures</b>	<b>Communication</b>
<b>Novice</b>	I do not understand the problem.	I am not sure how to do it.	I have no explanation. I'm not sure how to draw the problem, or which numbers to use.
<b>Apprentice</b>	I think I understand the problem.	I got started. I'm still thinking. I have part of the answer. It would help me to work with somebody.	I can explain some of what I did. I tried to use pictures, numbers, graphs, or words. My answer might be right.
<b>Practitioner</b>	I got it! I understand the problem.	I used a plan to solve the problem.	I can tell you or show you how I got the answer. I used mathematical terms and models, pictures, graphs, or numbers to explain how I did the problem. You can see how I did it.
<b>Expert</b>	I got it! I can prove to you that I'm right. I know I'm right.	I can show you several plans to solve this problem. I can prove to you that my answer is right.	I very clearly laid out step by step how I solved the problem. I used words, pictures, numbers, models, or graphs to show exactly how I did the problem, and proved that my answer is right. I can even show you another way to do the problem.

*How to Assess Problem-Solving Skills in Math* by Bena Kallick and Ross Brewer, Scholastic Professional Books, 1997

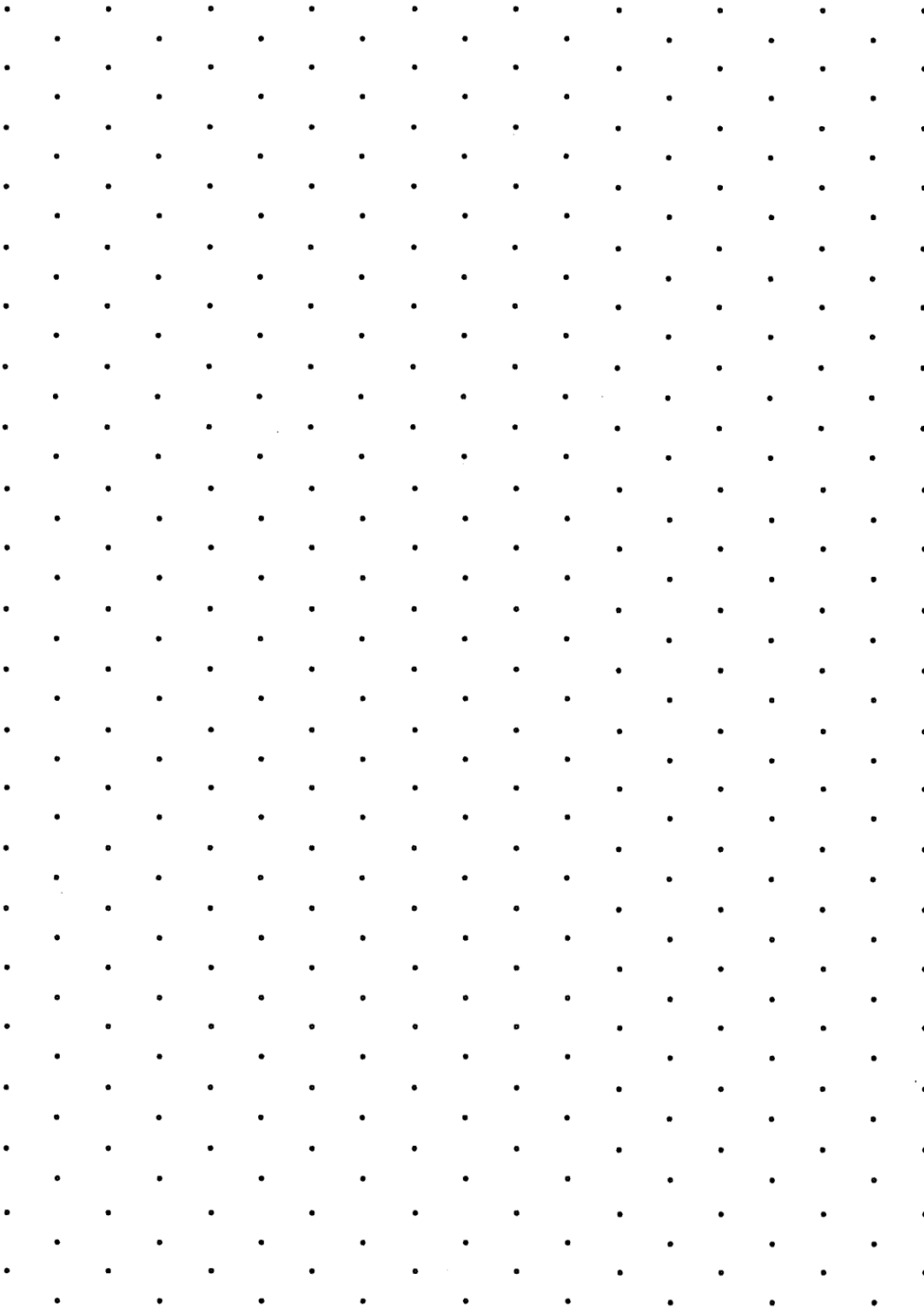
## 99 Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

# 100 Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Isometric Dot Paper



From *About Teaching Mathematics* © 1992 Math Solutions Publications

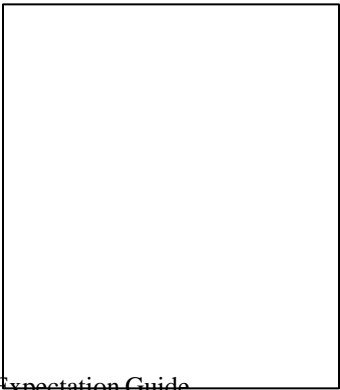
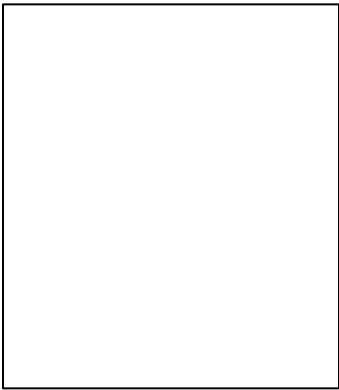
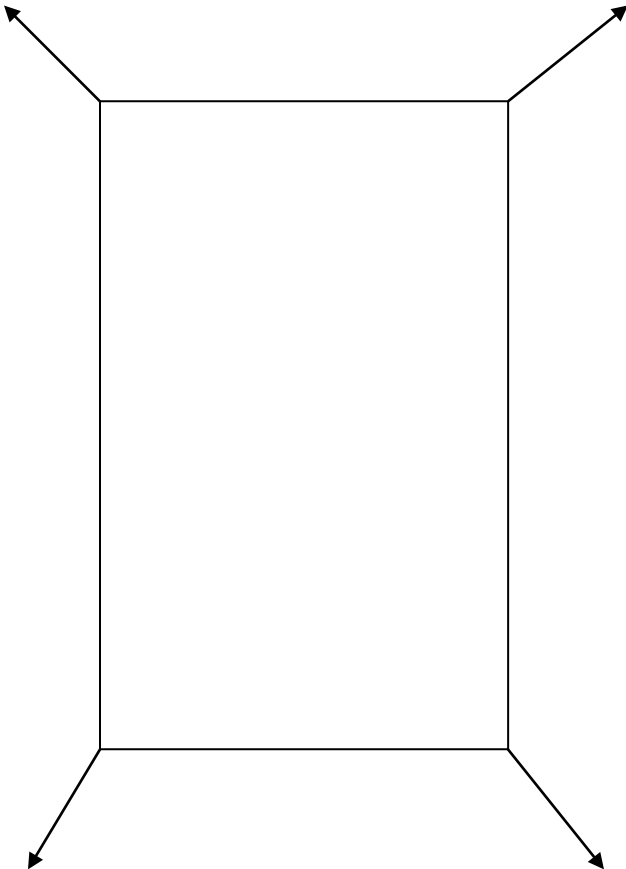
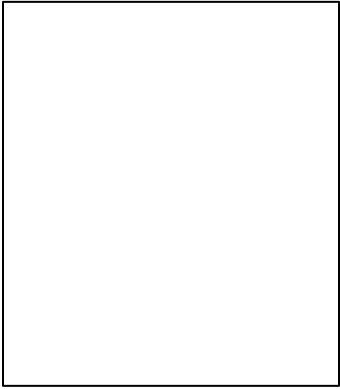
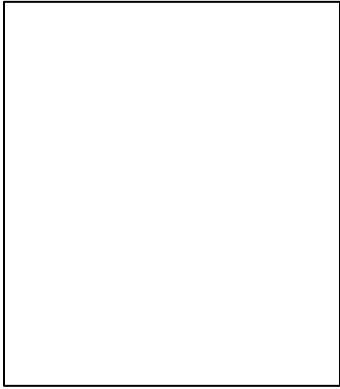
# TEN FRAME




# PART - PART - WHOLE MAT


# PART - PART - WHOLE MAT


**FLOW  
CHART**



# Place Value Mat

<b>Hundreds</b>	<b>Tens</b>	<b>Ones</b>

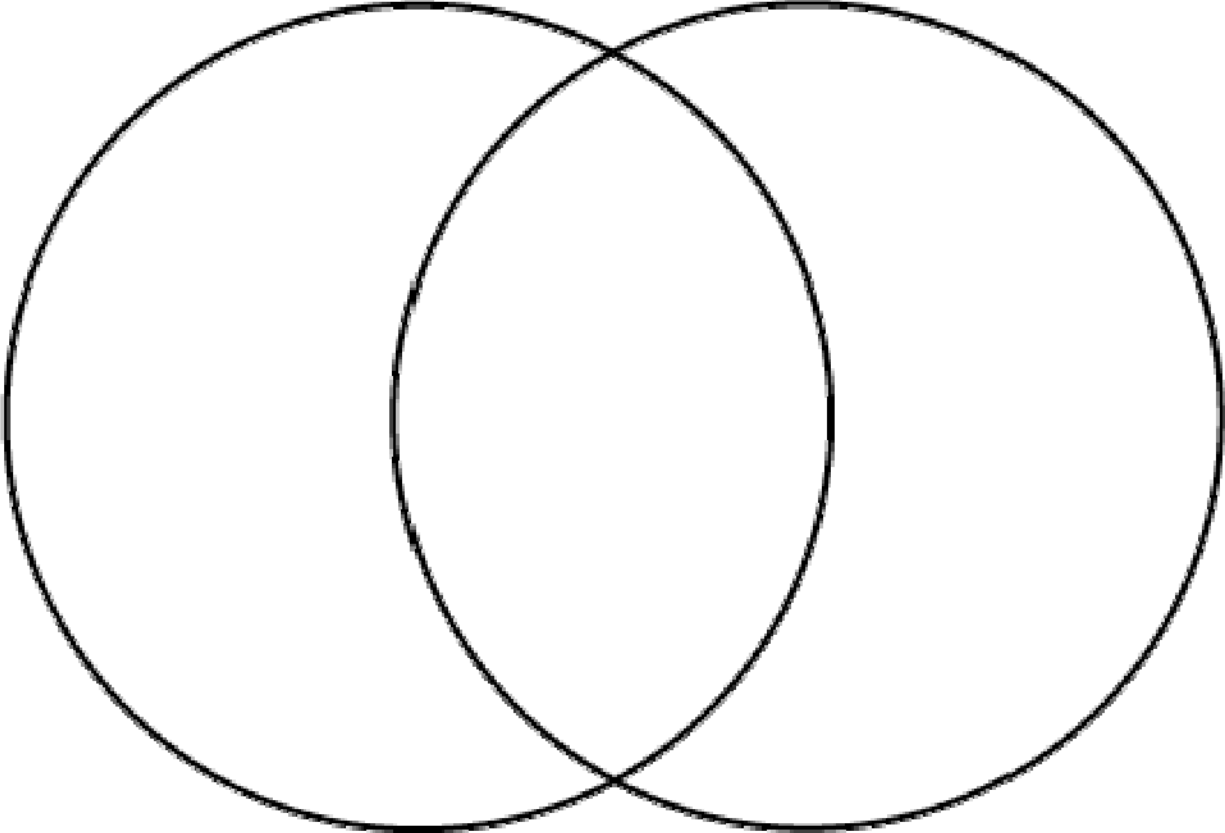
**Place Value Mat**

●	

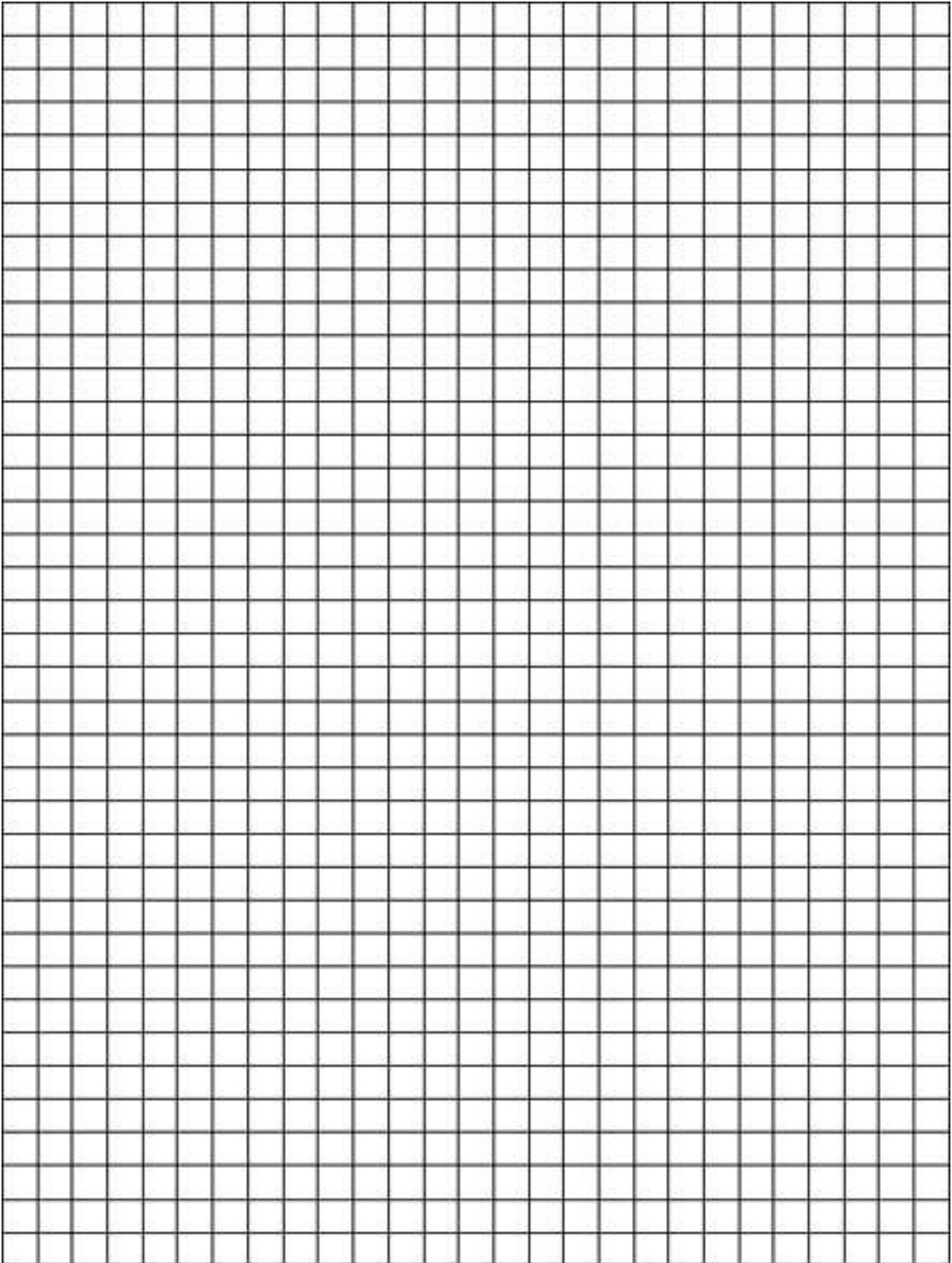
**Place Value Mat**


# Venn Diagram

Title:



What information did you learn or what conclusions can be drawn from having completed this Venn diagram?





# Math Websites

<http://ed.sc.gov/agency/offices/cso/mathematics/math.html> --- South Carolina State Department of Education (math content pages)

<http://ed.sc.gov/agency/offices/cso/instructionalresources.html> --- South Carolina State Department of Education (instructional resources)

<http://ed.sc.gov/topics/classroomresources/> --- Additional Classroom Resources

<http://www.khake.com/page47.html> --- Math Education Web Resources

<http://www.atozteacherstuff.com/themes/math> --- lesson plans

<http://www.amathsdictionary.com> --- teacher and student friendly dictionary of math terms

<http://www.funbrain.com/> --- math games

<http://www.math.com/> --- math lessons and practice problems

<http://www.mathforum.org/> --- teacher resources

<http://www.purplemath.com/> --- math assistance

<http://www.coolmath4kids.com/> --- math games

<http://mathcounts.org/> --- math counts (problem of the week)

<http://www.learner.org/teacherslab/math/geometry/> --- math activities

<http://arcytech.org/java/integers/> --- math lessons

<http://nlvm.usu.edu/en/nav/vlibrary.html> --- virtual manipulatives

<http://www.shodor.org/interactivate/activities/index.html> --- math activities

<http://www.nctm.org> and <http://illuminations.nctm.org/> --- (illuminations link) math activities

<http://school.discovery.com/teachingtools/teachingtools.html> --- lesson plans and student activities

<http://www.pbs.org/teachersource/math.htm> --- lesson plans and student activities

<http://www.goenc.com/> --- math and science activities

<http://www.mste.uiuc.edu/resources.php> --- math activities

<http://www.mathprojects.com/> --- lesson plans and activities

<http://msteacher.org/math.aspx> --- math activities

[www.donorschoose.org](http://www.donorschoose.org) --- obtain resources and materials

[www.e2c2.com/fileupload.asp](http://www.e2c2.com/fileupload.asp) --- For learning profiles to differentiate instruction, download the file entitled "Profile Assessments for Cards."

<http://education.ti.com> --- Texas Instrument website

<http://mathworld.wolfram.com/> --- math resources and links

<http://www.free.ed.gov/> ---- FREE, the website that makes teaching resources from federal agencies easier to find

<http://www.stat.uiuc.edu/courses/stat100/cuwu/> --- statistics (guess the correlation coefficient for a scatterplot)

<http://www.mathcats.com/mathtoolbox/index.html> --- everyday items as manipulatives

<http://www.learner.org/teacherslab/math/geometry/> --- space and shape in Geometry

<http://www.figurethis.org/> --- family outreach

<http://atschool.eduweb.co.uk/toftwood/resources.html> --- Interactive Games for Primary Grades

<http://www.cut-the-knot.org> - Interactive Mathematics Miscellany and Puzzles

<http://www.joma.org/> --- The Journal of Online Mathematics and its Applications

[http://enchantedmind.com/html/puzzles/interactive\\_puzzles/creative\\_java\\_puzzles.html](http://enchantedmind.com/html/puzzles/interactive_puzzles/creative_java_puzzles.html) --  
- Math Puzzles

<http://www.bbc.co.uk/education/mathsfiler/gameswheel.html> --- The Maths Files Game Show

<http://teacherlink.org/content/math/interactive/flash/top.html> --- Teacherlink: Interactive Mathematics Projects using Macromedia Flash

<http://www.math.uah.edu/stat/> --- virtual laboratories in probabilities and statistics

## Glossary of Terms

**Algorithm** - A specific set of instructions for carrying out a procedure or solving a problem.

**Array** - An arrangement (usually rectangular) of objects or numbers

**Base** - the face (of a polyhedron) or segment (of a polygon) that is being used as the reference to measure the height (altitude) of the polygon or polyhedron.

**Benchmarks** - Important units used as a referent for estimation. Benchmark numbers for fractions could be 0,  $\frac{1}{2}$ ,  $1$ ,  $1\frac{1}{2}$ , and so forth. Benchmark for measurements could be multiples of standard units. Benchmarks for whole numbers could be multiples of 10, 100, 1000, and so forth.

**Box Plot (Box-and-Whisker Plot)** - A representation of data with a rectangular box extending from the lower quartile to the upper quartile of the data and two lines extending from the ends of the box to the extreme values of the data.

**Cardinality of a Set** - When counting a set, the last number word used (how many in a set)

**Composing/Decomposing a Number** - A strategy used to reinforce number sense. Involves conceptualizing a number as being made up of two or more parts: putting the parts together to make a number is *composing* a number; breaking a number into two or more parts is *decomposing* the number.

**Compute Fluently** - Use efficient and accurate methods for computing.

**Cone** - a solid with a circular face and a point that is not in the same plane as the face

**Congruent** - having the same shape and same size

**Conjecture** – informed guessing

**Cylinder** - a solid with two congruent circular faces that are parallel and connected by a curved surface

**Dot Plot (Line Plot)** - A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

**Edge** - A line segment where two faces of a polyhedron meet.

**Faces** - The flat polygonal regions of a polyhedron.

**Geometric Pattern** - A pattern involving geometric shapes so that students see a pattern that involves square or triangular numbers; the pattern typically involves multiplication facts

**Histogram** - A special type of bar graph that displays the frequency of data as rectangles with areas proportionate to the corresponding frequencies. Each bar has the same width. The width of the bar represents a range of values along the horizontal axis.

**Inverse Relationship Between Operations** - The inverse of a mathematical operation undoes the operation. For example, subtraction undoes addition.

**Line Graph** - In a line graph, points representing two related pieces of data are plotted and then connected by a line.

**Line Plot (Dot Plot)** – A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

**Mean** - (a measure of central tendency) also known as the average, the sum of the values in a data set divided by the number of items in the data set

**Median** - (a measure of central tendency) the middle value of an ordered set of values

**Mode** - (a measure of central tendency) the score or data value in a set that occurs the most often

**Models**- Concrete, pictorial, symbolic, verbal, and algorithmic representations.

**Nets** - A two-dimensional fold-up model of a polyhedron.

**Networks** – A graph or directed graph together with a function that assigns a positive real number to each edge.

**Perfect Square** - The product of an integer multiplied by itself. For example, 4 is a perfect square because  $2 \times 2 = 4$ .

**Plane** (common notion) - a two dimensional surface that extends infinitely in all directions

**Polygon** - A closed plane figure with  $n$  sides. The sides of a polygon are line segments.

**Polygonal Regions** - Flat surfaces enclosed by polygons.

**Polyhedron** - A closed three-dimensional object whose surfaces are formed by polygonal regions (e.g. prism, pyramid, octahedron).

**Prism** - A polyhedron with two congruent, parallel bases that are polygons, and all remaining faces parallelograms.

**Pyramid** - A polyhedron with a polygon for a base and all other sides being triangles with one common vertex.

**Similar** - when two figures have the same shape and corresponding sides and angles are proportional

**Vertex** (of a polyhedron) - a point where three or more edges of a polyhedron meet

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<http://www.cast.org/ncac/index.cfm?i=2876> - This site contains an article by Tracy Hall at the National Center for Accessing the General Curriculum. The article discusses differentiation as it applies to the general education classroom.

<http://members.shaw.ca/priscillatheroux/differentiatingstrategies.html> - The Enhancing Learning with Technology site provides explanations for various differentiation strategies.

## **Enrichment Exemplar**

**Essential Question:**

**Enrichment criteria:**

**Student as investigator or mathematician:**

- Find and focus on problem
- Recognize significant information
- Categorize and critically analyze information
- Produce and effectively communicate results

**Problem situation base on current content:**

- Evidence of critical thinking
- Evidence of connection to real world, other subjects, students' interest or other math content
- Evidence of the process and results
- No evidence of pre-determined strategy or solution
- Designed by teacher or posed by student

**Grade Level:**

**Suggested materials:**

**Indicators addressed:**

**Standard:**

**Indicator:**



### **Kindergarten Enrichment Exemplar**

Is there a pattern between the length and the weight of measuring with non-standard tools? How do you know?

#### **Enrichment criteria:**

##### **Student as investigator or mathematician**

Find and focus on problem

Example: pattern between length and weight

Recognize significant information

Example: each measurement, difference between length and weight

Categorize and critically analyze information

Example: sort, analyze length and weight measurements

Produce and effectively communicate results

Example: student recording sheet with conclusions

##### **Problem situation base on current content**

Evidence of critical thinking

Example: sort, compare measurements

Evidence of connection to real world, other subjects, students' interest or other math content

Example: Number and operations (K-2.3), Data analysis and probability (K-6.1)

Evidence of the process and results

Example: student recording sheet, teacher observation- see students measuring with tools, working collaboratively, etc.

No evidence of pre-determined strategy or solution

Example: patterns and representations of results vary due to tools and objects used by students

Designed by teacher or posed by student

Example: modified from Thinkcentral.com Enrichment Workbook with Projects

#### **Kindergarten:**

Is there a pattern between the length and the weight of measuring with non-standard tools? How do you know?

#### **Suggested materials:**

paper clips, cm unit cubes, unifix cubes, etc., balance scale, student recording sheet

#### **Indicators addressed:**

**Standard K-5:** The student will demonstrate through the mathematical processes an emerging sense of coin value and the measurement concepts of length, weight, time, and temperature.

#### **Indicator:**

**K-5.3 :** Use non standard unit to explore the measurement concept of length and weight.

## First Grade Enrichment Exemplar

### Essential Question:

What is the importance of predictable patterns? Can visual representations be created from a numerical pattern and does the same relationship exist?

### Enrichment criteria:

#### Student as investigator or mathematician:

Find and focus on problem

Example: determine what comes next

Recognize significant information

Example: number recognition, as numbers are listed, they are increasing or decreasing in amounts

Categorize and critically analyze information

Example: analyze the relationship between numbers (did the numbers increase/ decrease, by how many, what change did you notice?)

Produce and effectively communicate results

Example: student's visual display or picture

#### Problem situation base on current content:

Evidence of critical thinking

Example: analyze, determine the next sequence in the pattern, be able to continue the pattern, create a visual representation of the numerical pattern, and explain how they know that it is pattern

Evidence of connection to real world, other subjects, students' interest or other math content

Example: Numbers and Operations (1-2.6), Algebra (1-3.5)

Evidence of the process and results

Example: student display, teacher observation, students collaborating

No evidence of pre-determined strategy or solution

Example: patterns and representations of results vary due to tools and objects used by students

Designed by teacher or posed by student

Example: modified from essential questions grade 1 Math

**First Grade:** What is the importance of predictable patterns? Can visual representations be created from a numerical pattern and does the same relationship exist?

**Suggested materials:** wipe boards, geometric shapes, paper, inch graph paper, coins, dice, dominos

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=35>

### Indicators addressed:

**Standard 1-3:** The student will demonstrate through the mathematical process a sense of numeric patterns, the relationship between addition and subtraction, and change over time.

#### Indicator:

**1-3.4:** Analyze numeric relationships to complete and extend simple patterns

## Second Grade Enrichment Exemplar

**Essential Question:** How do strategies assist in more efficient and accurate computation?

### Enrichment criteria:

#### Student as investigator or mathematician:

Find and focus on problem

Example: develop strategies and accuracy

Recognize significant information

Example: counting, adding, and subtracting

Categorize and critically analyze information

Example: create and apply strategies

Produce and effectively communicate results

Example: student recording sheet or journal with conclusions

#### Problem situation base on current content:

Evidence of critical thinking

Example: create strategies using multiple tools

Evidence of connection to real world, other subjects, students' interest or other math content

Example: Number and Operations (2-2.4, 2-2.10); Use data (numbers) from science, PE, lunch calories, or another subject

Evidence of the process and results

Example: student work.

No evidence of pre-determined strategy or solution

Example: strategies will vary due to tools and knowledge level of student understanding; if students use data from PE or lunch, most of their numbers will be different which makes the solutions different

Designed by teacher or posed by student

Example: modified from essential questions

## Second Grade

**Suggested materials:** white board, base-ten blocks, square tiles, other counting tools, paper pencil

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=8>,

[http://nlvm.usu.edu/en/nav/category\\_g2t1.html](http://nlvm.usu.edu/en/nav/category_g2t1.html)

### Indicators addressed:

**Standard: 2-2:** The student will demonstrate through the mathematical process an understanding of the base-ten numeration system; place value; and accurate, efficient, and generalizable methods of adding and subtracting whole numbers.

#### Indicator:

**2-2.7:** Generate strategies to add and subtract pairs of two-digit whole numbers with regrouping.

### **Enrichment Third Grade Exemplar**

**Essential Question:** How do understanding patterns, sequences, and functions help us to solve problems?

#### **Enrichment criteria:**

##### **Student as investigator or mathematician:**

Find and focus on problem

Example: find patterns, determine the sequence

Recognize significant information

Example: types of patterns

Categorize and critically analyze information

Example: analyze pattern and sequence changes in the pattern

Produce and effectively communicate results

Example: student recording sheet or journal with conclusion

##### **Problem situation base on current content:**

Evidence of critical thinking

Example: analyze to determine pattern, sequence, and function

Evidence of connection to real world, other subjects, students' interest or other math content

Example: Numbers and operations (3-2.3, 3-2.7, 3-2.9); create and analyze cultural patterns in Social Studies and Art, and Animal, Rock /Mineral, and Regional patterns in Science (to include increase of change over time)

Evidence of the process and results

Example: student creations and collaboration

No evidence of pre-determined strategy or solution

Example: pattern and representations of results may vary

Designed by teacher or posed by student

Example: modified from Thinkcentral.com Enrichment Workbook with Projects

#### **Third Grade:**

How do understanding patterns, sequences, and functions help us to solve problems?

#### **Suggested materials:**

Pattern blocks, square tiles, grid paper, snap cubes, math journals or recording sheet

#### **Indicators addressed:**

**Standard 3-3:** The student will demonstrate through the mathematical processes an understanding of numeric patterns, symbols as representations of unknown quantity, and situations showing increase over time

**Indicators: 3-3.1** Create numeric patterns that involve whole number operations

**3-3.3** Use symbols to represent an unknown quantity in a simple addition, subtraction, or multiplication equation

**3-3.4** Illustrate situations that show change over time as increasing

### **Enrichment Fourth Grade Exemplar**

**Essential Question:** What are ways that parts of a whole (fractions) are represented in everyday life?

#### **Enrichment criteria:**

##### **Student as investigator or mathematician:**

Find and focus on problem

Example: find fractions, create equivalent fractions and compare fractions

Recognize significant information

Example: determine the relationship between the numerator and denominator

Categorize and critically analyze information

Example: compare fractions, determine which is closest to 0,  $\frac{1}{2}$ , 1

Produce and effectively communicate results

Example: math journals with conclusions

##### **Problem situation base on current content:**

Evidence of critical thinking

Example: create visual presentations of fractions; re-represent units to create new fractions

Evidence of connection to real world, other subjects, students' interest or other math content

Example: Numbers and Operations 4-2.3, Algebra 4-3.2, create fractions using food

Evidence of the process and results

Example: student creations and collaboration

No evidence of pre-determined strategy or solution

Example: fractions, comparisons, and representations may vary

Designed by teacher or posed by student

Example: modified from Thinkcentral.com Enrichment Workbook with Projects

**Fourth Grade:** What are ways that parts of a whole (fractions) are represented in everyday life?

**Suggested materials:** fractions tiles, bars, circles, Cuisenaire rods, geometric shapes, snap cubes, square tiles, grid paper

#### **Indicators addressed:**

**Standard:** The student will demonstrate through the mathematical process as understanding of decimal notation as an extension of the place-value system; the relationship between fractions and decimals; the multiplication of whole numbers; and accurate, efficient, and generalization methods of dividing whole numbers, adding decimals, and subtracting decimals.

**Indicator: 4-2.8** Apply strategies and procedures to find equivalent forms of fractions.

**4-2.9** Compare the relative size of fractions to the benchmarks  $0, \frac{1}{2}, 1$ .

## 5<sup>th</sup>/6<sup>th</sup> Grade Enrichment Exemplar

Using pattern blocks, we can build a polygon "chain" that consists of 2 equilateral triangles and 3 trapezoids.



- A. Describe the train in as many ways as possible.  
Possible answers are: Encourage the students to go beyond the obvious.

+  $\frac{3}{5}$  of the chain is a trapezoid

+  $\frac{2}{5}$  of the chain is a triangle

+ The ratio of triangles to trapezoids is 2:3

- B. If you have 4 triangles and 6 trapezoids, how many "chains" can you build?  
Sketch the chains below.

- C. Complete the table below and answer the questions that follow.

Number of Chains	Number of Triangles	Number of Trapezoids	Ratio of Triangles to Trapezoids
1	2	3	2:3
2			
3			
4			
		18	
			32:48
	135		

Do you notice any patterns in the table? What are they?

- D. If the number of trapezoids used to make "chains" is 18, how many triangles are needed?  
What is the ratio of triangles to trapezoids?  
What is the relationship between this ratio and the ratio of triangles to trapezoids pictured in Part A?

How do you know?

- E. If you want to use 135 triangles, how many trapezoids will you need so the relationships between triangles and trapezoids is maintained?

- F. If the total number of blocks is 65, how many triangles and trapezoids are there?
- G. Represent your solution.

**Enrichment criteria:**

**Student as investigator or mathematician**

- + Find and focus on problem  
Example: determining the relationship between the number of triangles and trapezoids or making a comparison between the number of triangles and trapezoids
- + Recognize significant information  
Example: a pattern exists
- + Categorize and critically analyze information  
Example: using the table and the pictorial representation to compare the number of triangles and trapezoids
- + Produce and effectively communicate results  
Example: **chart with results, manipulation of concrete tools, oral presentation of results**

**Problem situation base on current content**

- + Evidence of critical thinking  
Example: reasoning through the patterns in the table to complete the table and answer the discussion questions
- + Evidence of connection to real world, other subjects, students' interest or other math content  
Example: connection to Algebra (patterns): **Indicator 6-3.1** Analyze numeric and algebraic patterns and pattern relationships
- + Evidence of the process and results  
Example: students work and pictures
- + No evidence of pre-determined strategy or solution  
Example: students' work should show their own thinking, interpretation and representations
- + Designed by teacher or posed by student  
Example: designed by the teacher

**6<sup>th</sup> Grade:**

**Suggested materials:** pattern blocks, calculator

**Indicators addressed:**

**Standard 6-2:** The student will demonstrate through mathematical processes and understanding of the concepts of whole-number percentages, integers, and ratio and rate; the addition and subtraction of fractions; accurate, efficient, and generalizable methods of multiplying and dividing fractions and decimals; and the use of exponential notation to represent whole numbers.

**Indicator:**

**6-2.6 :** Understand the relationship between ratio/rate and multiplication /division.

**Note:**

- Y Students may believe that all ratios are fractions. This is not true because the difference is that fractions always represent part-to-whole relationships. On the other hand, ratios can represent part-to-whole OR part-to-part relationships.
- Y It is important to include the quantity names in order to be clear about exactly what is being compared.

**Possible Solution:**



**A. Describe the train in as many ways as possible.**

Possible answers are: Encourage the students to go beyond the obvious.

- +  $\frac{3}{5}$  of the chain is a trapezoid
- +  $\frac{2}{5}$  of the chain is a triangle
- + The ratio of triangles to trapezoids is 2:3

**B. If you have 4 triangles and 6 trapezoids, how many “chains” can you build?**  
2 Sketch the chains below.

**C. Complete the table below and answer the questions that follow.**

Number of Chains	Number of Triangles	Number of Trapezoids	Ratio of Triangles to Trapezoids
1	2	3	2:3
2	4	6	4:6
3	6	9	6:9
4	8	12	8:12
6	12	18	12:18
16	32	48	32:48
45	90	135	90:135

Do you notice any patterns in the table? What are they? The number of triangles is increasing by 2 and the number of trapezoids always increase by 3 as you go down the table. The total of the shapes is always a multiple of 5 and based on the number of chains.

**D.** If the number of trapezoids used to make "chains" is 18, how many triangles are needed? 12  
 What is the ratio of triangles to trapezoids? 12:18  
 What is the relationship between this ratio and the ratio of triangles to trapezoids pictured in Part A? The ratio 12:18 is a multiple of the ratio in Part A.  
 How do you know? I can multiply each part of the ratio in A by 6 to get the ratio 12:18.



- H. If you want to use 135 trapezoids, how many triangles will you need so the relationships between triangles and trapezoids is maintained? 90
- I. If the total number of blocks is 65, how many triangles and trapezoids are there?  
There would be 13 chains, so  $2 \bullet 13 = 26$  triangles and  $3 \bullet 13 = 39$  trapezoids.  
The total number of blocks is  $26 + 39 = 65$ .
- J. Represent your solution.

**Resources:**

Math Matters (2<sup>nd</sup> edition). By Suzanne H. Chapin and Art Johnson (p. 168 - 172)